APPLICATION of the 3-WAY DEDICOM MODEL to SKEW-SYMMETRIC DATA for PAIRED PREFERENCE RATINGS of TREATMENTS for CHRONIC BACK PAIN

by

Margaret E. Lundy[1], Richard A. Harshman[2], Pentti Paatero[3],

and Leora C. Swartzman[4]

[1, 2, 4] Psychology Department, University of Western Ontario, London, Canada
[3] Physics Department, University of Helsinki, Helsinki, Finland

[2, 4] Research funded by the Canadian Institute of Health Research (MOP-44037) and the University of Western Ontario Development Fund ([4] is principal investigator; [2] is co-investigator)
[2] Research funded by the Natural Sciences and Engineering Research Council of Canada (RGPIN 7896-01)

TRICAP 2003, Lexington, Kentucky, June 2003

Abstract

A 3-way generalization of the DEDICOM model (Decomposition into Directional COMponents) for skew-symmetric data (Harshman & Lundy, 1990) was applied to student paired preference ratings of 19 different treatments for chronic back pain. The model fitting process was accomplished using the Multilinear Engine program (Paatero, 1999). By imposing various constraints on the model during the data analysis, 3 distinct preference hierarchies amongst the treatments were identified: one amongst psychological treatments and herbal remedies, one amongst conventional medical treatments, and one amongst complementary/ alternative (CAM) physical treatments. The direction of preference within these hierarchies may be reversed for some people. Theoretical and practical implications are discussed.

The DEDICOM Model for Skew-Symmetric Preference Data

Definitions

DEDICOM: **DE**composition into **DI**rectional **COM**ponents
Skew-symmetric matrix: • diagonals are zero **and**
• off-diagonals are symmetric in value

but opposite in sign
Bimension: A preference (dominance) hierarchy defined by **2** factors (e.g., Bimension 1 is defined by factors 1 and 2, Bimension 2 is defined by factors 3 and 4, etc.)

Model: Given an *m* by *m* by *p* array $\underline{X}$ of pairwise preference comparisons amongst *m* stimuli made by *p* raters, the 3-way DEDICOM model for *r* bimensions (2*r* factors) may be written as:

$$\mathbf{X}_k = \mathbf{A}\ \mathbf{D}_k\ \mathbf{H}\ \mathbf{D}_k\ \mathbf{A}' + \mathbf{E}_k \qquad (1)$$

where

- $\mathbf{X}_k$ is the *k*th *m* by *m* slice of $\underline{X}$ *and* is **skew-symmetric** in form (i.e., $x_{iik}=0$ for $i=1, 2, \ldots, m$ and $x_{ijk} = -x_{jik}$ for $i, j=1, 2, \ldots, m$ and $i \neq j$)
- **A** is an *m* by 2*r* matrix of **stimulus factor weights**
- $\mathbf{D}_k$ is a *2r* by 2*r* diagonal matrix, where the diagonal entries are from the *k*th row of **C**, a *p* by 2*r* matrix of **person factor weights**, *and* $d_{(i-1)(i-1)k} = d_{iik}$ for $i=2, 4, \ldots, 2r$ (i.e., pairs of diagonal values – and columns in **C**– are equal)
- **H** is a 2*r* by 2*r* **skew-symmetric** matrix of **interactions** amongst the factors in A; additional restrictions on the form are given below
- $\mathbf{E_k}$ is an *m* by *m* matrix of random error

Model (1) is the 3-way extension of the 2-way model presented in Harshman & Lundy (1990).

Additional restrictions on H

Each 2 by 2 **between**-bimension interaction block is restricted to have only 2 distinct values, with one sign difference, of the form

$$\begin{matrix} c & d \\ -d & c \end{matrix}$$

where c, d can take on any real value.

Thus the 2-bimensional (4 factor) **H** has the form

		Bimension 1		Bimension 2	
	factor	1	2	3	4
Bim 1	1	0	a	**c**	**d**
	2	-a	0	**-d**	**c**
Bim 2	3	**-c**	**d**	0	b
	4	**-d**	**-c**	-b	0

Comparison with 3-way factor analysis

A: • **bimensions** rather than factors are interpreted
- a 2-way plot of stimulus factor weights on, e.g., factors 1 and 2, illustrates the preference hierarchy defined by bimension 1
- even number of factors in A follows from the even-rank property of skew-symmetric matrices in general

C: • **pairs of equal columns** or person weights, rather than independent weights, due to the bimensional nature of the model (but see relaxed restriction below)

H: • bimension interactions are analogous to angles between oblique factors, but are scaled here to give both the **strength** of each preference hierarchy (a and b in **H** above) and the preference of one bimension over another (c and d in **H** above). **Direction of preference** is given by the sign.

Further Generalization of the Model

Model (1) restricts the hierarchies to have the same direction of preference for all Ss, but this may often be inappropriate. In this application, the pairwise equality restriction on the diagonals in $\mathbf{D}_k$ and the columns of C is relaxed so that

$$d_{(i-1)(i-1)k} = \pm\, d_{iik} \quad \text{for } i=2, 4, \ldots, 2r \tag{2}$$

(i.e., equal absolute value).

The Preference Data

- **Stimuli**: 19 treatment/management approaches for chronic back pain (see Table 1 below)
- **Subjects**: 115 undergraduate psychology students (75 females; 40 males; mean age=20) at the University of Western Ontario who participated as part of their course requirements
- **Task**: Rate pairs of treatments on a 5-point scale in terms of the strength of one's preference for one over the other as a management approach for chronic back pain. All distinct pairs of treatments are rated (i.e., 184 pairs).

Example Item

	Very Strong Preference				No Preference				Very Strong Preference	
Massage Therapy	**4**	**3**	**2**	**1**	**0**	**1**	**2**	**3**	**4**	**Back Surgery**

A subject would circle 4 to the left of zero if s/he had a very strong preference for massage over surgery or would circle 1 to the right of zero if s/he a slight preference for surgery over massage.

Data array: 115 19 by 19 matrices of skew-symmetric data. Positive numbers indicate row treatment is preferred over column treatment; negatives indicate the column treatment is preferred over the row. Zeroes are on the diagonal and elsewhere where preference ratings are neutral.

Example Matrix

treatments	1. 13	**surgery**	**massage**	16. 19
1. 13		:		
surgery	. . . .	**0**	**- 4**	. . . .
massage		**4**	**0**	
16. 19		:		

Data from the example item above would be entered like this if the subject circled 4 to the left of zero.

Table 1. Preference Rating Task Stimuli

1. **Prescription pain killing tablets (non-narcotic)**
2. **Significant weight loss**
3. **Chiropractic adjustments**
4. **Herbal teas**
5. **Prescription cream applied to affected area**
6. **Psychological treatments**
7. **Acupuncture**
8. **Non-prescription (over the counter) pain killing tablets**
9. **Meditation**
10. **Homeopathic medicine (in tablets) (i.e., extremely small doses of a substance that would, in healthy people, produce symptoms of the disease being treated)**
11. **Physical exercise program**
12. **Prescription pain killing tablets (containing narcotics)**
13. **Hypnosis**
14. **Back surgery (requiring hospital stay)**
15. **Massage therapy**
16. **Nutriceutical tablets (containing large doses of certain vitamins, minerals, or other nutritional supplements)**
17. **Using one’s inner spiritual strength**
18. **Injections of pain killing medication into affected area**
19. **Herbal tablets**

Analysis and Results

- Multilinear Engine (Paatero, 1999) program used to fit model

- Preliminary analyses identified 3 distinct clusters, each with some preference ordering of the treatments contained therein, but they did not appear on separate bimensions

- Additional constraints imposed to “guide” final analysis:
  - pull-to-zero on **H** elements to prevent divergent estimates associated with degenerate solutions
  - quasi-orthogonal constraints on factors in **A** by forcing the 3 previously identified nonoverlapping groups of treatments to be fit by different factor pairs (bimensions)
  - 3-bimensional fit: $r^2 = 0.29$, rmse = 2.11, stress = 0.84

- **Plots** of 3 bimensions (Fig. 1–3) using **A** factor pairs illustrate fairly linear preference hierarchies amongst, respectively,
  1. psychological treatments and herbal remedies
  2. conventional medical treatments
  3. physical complementary/alternative therapies

  Preference direction is top to bottom (higher points preferred over lower points) for Ss whose two person weights in **C** for the bimension have the same sign, but bottom to top for Ss with oppositely-signed person weights.

- **H interactions** (see below) show:
  - **within-bimension** weights are substantially larger for bimension 2 (0.25) and approximately equal for 1 and 3 (0.14, 0.15); this suggests that bimension 2 accounts for more variance than the others
  - some much larger **between-bimension** weights (0.40, 0.53) suggest that preferences between bimensions account for even more variance

H interaction matrix

	Bimension 1		Bimension 2		Bimension 3	
factor	1	2	3	4	5	6
1	0	**0.14**	0.03	0.26	-0.03	0.53
2	-0.14	0	-0.26	-0.03	-0.53	-0.03
3	--0.03	0.26	0	**0.25**	-0.04	0.40
4	-0.26	0.03	-0.25	0	-0.40	-0.04
5	0.03	0.53	0.04	0.40	0	**0.15**
6	-0.53	0.03	-0.40	0.04	-0.15	0

- A more detailed examination of the results is beyond the scope of this presentation.

Summary Remarks

- First attempt (that we know of) to fit the 3-way DEDICOM model defined by (1) and (2) to paired preference ratings met with mixed success. Apparently much variance in the data is not appropriate for the model and so additional constraints, based on results of unsuccessful analyses, were imposed to get a clearer solution. Nonetheless, the 3 preference hierarchies so obtained are quite interpretable and informative.

- It may be that these subjects are heterogeneous not only with respect to their direction of preference within treatment groups (which can be accommodated by (2)), but also with respect to their preference for one bimension over another (which cannot). For this model, the only way to control for problems arising from different preferences between bimensions is to force groups of treatments to be fit by separate bimensions, as was done in the "guided" analysis.

- It may be that similar (if not so severe) problems may be encountered with most types of preference data when fitting this model, because of differences across groups of individuals.

- This model is not restricted to preference data, but may be applied to any skew-symmetric data for which directional hierarchies amongst stimuli are expected (e.g., skew-symmetric part of trade data, or perhaps some sort of chemical transformation data). In these cases, a bimension would define a "dominance hierarchy" rather than a preference hierarchy. One might expect a more straightforward application of (1) for such data.

References

Harshman, R. A., & Lundy, M. E. (1990). Multidimensional analysis of preference structures. In A. deFontenay, M. H. Shugard, & D. S. Sibley (Eds.), *Contributions to ecomomic analysis: Vol. 187. Telecommunications demand modeling: An integrated view* (pp. 185-204). Amsterdam: Elsevier.

Paatero, P. (1999). The multilinear engine–A table-driven, least squares program for solving multilinear problems, including the n-way parallel factor analysis model. *Journal of Computational and Graphical Statistics*, *8*, 854-888; computer software and user manual was retrieved from ftp://rock.helsinki.fi/pub/misc/pmf/me2 Jan 2003.

Figure 1. Bimension 1 of 3-bimensional "guided" analysis

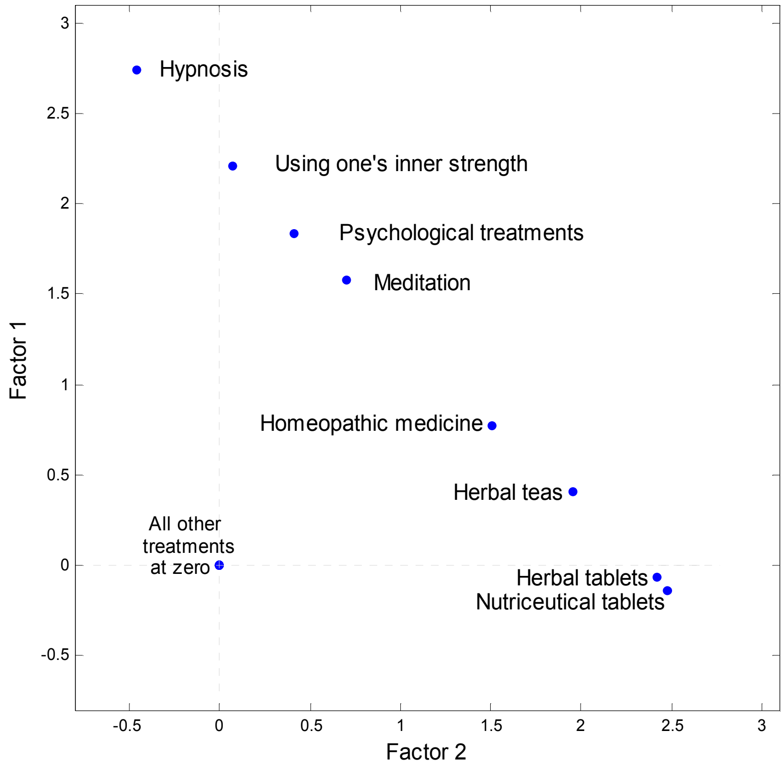

Figure 2. Bimension 2 of 3-bimensional "guided" analysis

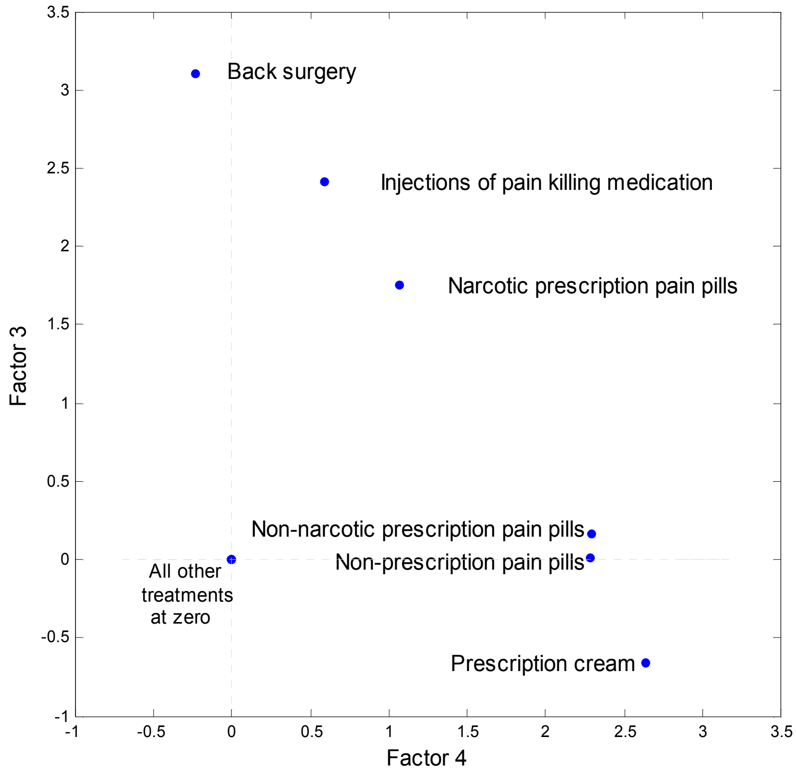

Figure 3. Bimension 3 of 3-bimensional "guided" analysis