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MULTIDIMENSIONAL ANALYSIS OF PREFERENCE STRUCTURES*

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A novel approach is proposed for analysis of paired-comparison preference data. Instead of fitting a specific psychological model, an exploratory analysis method called DEDICOM can be used to obtain a multidimensional representation of preferences much like the representation that factor analysis or multidimensional scaling provides for other kinds of data. To demonstrate the method, it is applied to an 8 by 8 matrix giving strength of preferences among foods, and a 9 by 9 matrix of preference choice frequencies among celebrities. Both analyses yield interpretable "dimensions" of stimulus preference or utility. They also demonstrate how DEDICOM can provide (a) simple linear or nonlinear preference scales, when appropriate; (b) multiple "dimensions" of preference when required by the data; (c) scale values for the stimuli on each scale; and (d) increments in the variance accounted for resulting from each additional "dimension" of preference or stimulus utility. Exploratory and psychological-model-based approaches are compared, and two possible explanations for the multidimensionality of preferences are discussed.

1. INTRODUCTION

It is often assumed that preference relationships among a set of objects can be represented by ordering the objects along a single line or dimension, corresponding to the "utility" of the stimuli. One can force a subject's data to have this type of structure by asking him/her to order the objects from most to least preferred. But if instead the subject is asked to independently assess all the pairwise relationships among the stimuli (e.g., by the method of "graded paired comparisons"), the resulting data often seem too complex to be produced by any ordering along a single dimension.

One of two approaches might be taken to investigating these complexities. The first is theory-based, and involves construction of special purpose models which incorporate psychological effects such as modulation of preference strength by stimulus similarity, or hierarchical choice. By fitting these models to observed preference data, one can test how well such

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effects account for the complexities, and one can also estimate the "underlying" preferences that the stimuli would elicit if the extra psychological effects were not present. This is the approach taken by most current investigators (e.g., [1]; [2]; [3], section III; [4]; [5]; cf. [6]) and is well exemplified by DeSoete and Carroll's contribution to this volume [7].

The second approach is more descriptive and "data-analytic". It attempts to decompose the complexities by some general purpose analytical procedure similar to multidimensional scaling, rather than theoretically "accounting for" and possibly eliminating them. Recall, for example, how theoretical models often try to account for the "multidimensionality" of preference by first constructing a multidimensional "attribute space" that is evaluatively neutral, and then defining a preference rule which operates on that space. In contrast, DEDICOM constructs no neutral attribute space. It decomposes the preferences themselves, and constructs evaluative "dimensions" which span a preference structure that is only quasi-spatial. One can then examine the obtained preference dimensions to see what kind of psychological theory they might suggest. In more applied situations, one can also use the extracted dimensions or patterns for marketing and forecasting purposes (e.g., to suggest important patterns of judgment underlying complex product evaluations, to infer which stimulus attributes affect preference, or to predict preferences for objects not in the stimulus set).

While interesting data-analytic methods had been developed for certain other kinds of data such as preference rankings (see [1] and [3] for excellent discussions), until recently the proper mathematical tools did not exist for matrices of paired comparisons. Traditional methods of uncovering latent multidimensional structure in a table of relationships, such as factor analysis or multidimensional scaling, all require that the relations be symmetric (as with "correlation" or "similarity"); preferences, on the other hand, are strongly asymmetric. Indeed, preference relationships tend to be anti- or "skew-symmetric" (i.e., pref(a over b) = -pref(b over a)). Recently, several methods for scaling asymmetric relationships have been proposed (e.g., [8]; [9]; [10]; [11]; [12]; [13]); the most general of these is DEDICOM (for DEcomposition into DIrectional COMponents; see [14] and [15]). The following sections illustrate the DEDICOM data-analytic multidimensional approach for discovering the structure of paired comparison preference data.

2. THE DEDICOM PROCEDURE

DEDICOM is a procedure for multidimensional analysis of matrices of relationships. It can be applied to real-valued matrices of any form: symmetric (where $x_{ij}=x_{ji}$); general asymmetric (where x_{ij} might not equal x_{ji}); or skew-symmetric (where $x_{ij}=-x_{ji}$). We restrict discussion here to the special case of skew-symmetric data, since this is the form taken by pairwise preference relationships. (The two-way DEDICOM representation of general asymmetric data is briefly described in Appendix A.) We show how the DEDICOM model directly decomposes a table of pairwise preferences into a few basic preference patterns, each of which can be attributed to one "dimension" or aspect of utility (instead of first finding a stimulus attribute space and then deriving utilities from directions or locations in this space, as most other models do). To facilitate interpretation, we present the component patterns graphically. "Scale values" for particular stimuli can also be determined by taking projections onto straight or curved lines, but because of space restrictions this is not shown.

2.1. The DEDICOM Model for Skew-symmetric Data

Let $\mathbf{X}$ be a matrix of pairwise preference relationships, with the entry x_{ij} describing the amount that alternative i is preferred over alternative j (a negative preference value is used if j is actually preferred over i). In such a matrix, x_{ij} is usually (approximately) equal to $-x_{ji}$, that is, $\mathbf{X}$ is usually (approximately) skew-symmetric. Here we assume that any deviations from skew-symmetry are due to measurement error; if the data are not perfectly skew-symmetric, we compute their skew-symmetric part by taking $0.5(\mathbf{X} - \mathbf{X}')$.

The DEDICOM representation of skew-symmetric $\mathbf{X}$ has the form

$$(1)\quad \mathbf{X} = \mathbf{A}\,\mathbf{R}\,\mathbf{A}' + \mathbf{E}$$

where $\mathbf{A}$ is an n by s matrix of "loadings" of the n stimuli on s dimensions (for even s only), $\mathbf{R}$ is a skew-symmetric s by s matrix of relationships among the dimensions, and $\mathbf{E}$ is an n by n matrix of residual error terms. The model is fit by least squares. For skew-symmetric $\mathbf{X}$, this is accomplished by taking the singular value decomposition of X and transforming it to appropriate form (see Appendix B for the details).

Note that we specified for (1) that the number of dimensions be even. This is a consequence of the well-known "even rank" property of skew-symmetric matrices (see, e.g., [16]). That is, two vectors or "dimensions" are required to generate even the simplest skew-symmetric matrix; more complex matrices may require four, six, or more component vectors, but the number required is never odd. Thus, any complex skew-symmetric matrix is represented as the sum of elementary rank-2 skew-symmetric matrices rather than rank-1 components as in factor analysis. Taken pairwise, DEDICOM dimensions (i.e., columns of $\mathbf{A}$) generate such rank-2 matrices. $\mathbf{X}$ is approximated by weighting each of these matrices by the appropriate value from $\mathbf{R}$ and then summing the weighted matrices.

For example, consider a four-dimensional DEDICOM solution. Let $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ be the columns of $\mathbf{A}$. Taking them pairwise, we can generate two rank-2 skew-symmetric matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ such that $\mathbf{K}_1 = \mathbf{a}_1\mathbf{a}_2' - \mathbf{a}_2\mathbf{a}_1'$ and $\mathbf{K}_2 = \mathbf{a}_3\mathbf{a}_4' - \mathbf{a}_4\mathbf{a}_3'$. Weighting $\mathbf{K}_1$ and $\mathbf{K}_2$ by elements of $\mathbf{R}$, the "orthogonal" (explained below) representation is then $X = r_{12}\mathbf{K}_1 + r_{34}\mathbf{K}_2 + \mathbf{E}$.

The even-rank property of skew-symmetric matrices has important consequences for DEDICOM analysis. First, DEDICOM must deal with pairs of dimensions in much the same way that traditional factor analysis deals with single axes or dimensions. For example, DEDICOM rotates two pairs of dimensions at a time rather than two axes. Second, we do not interpret each individual dimension but rather the plane defined by each pair of dimensions. We view the plane as representing the preference pattern contributed by one kind of utility underlying the stimulus preferences. (Interpretation of single dimensions is not prohibited, but is usually not done for the skew-symmetric case. What we are interested in is dominance patterns, which are revealed by the planes; this is discussed below.) Finally, we use the term "bimension" to refer to the plane defined by two dimensions (i.e., the four-dimensional solution above is said to be two-bimensional). Hereafter, "bimension" will be used to mean the plane defined by a pair of DEDICOM dimensions.

2.2. Transformations or "Rotations"

Immediately after the singular value decomposition of X (see Appendix B), the bimensions are like principal axes, mutually orthogonal and oriented so that each successive bimension accounts for as much of the remaining variance as possible. These initial "unrotated" bimensions usually represent linear combinations of the preference patterns arising from several different kinds of utility. More interpretable bimensions, corresponding to the (partial) preferences due to individual "kinds of utility" or aspects of preference, can often be obtained by means of axis "rotations" similar to those used in factor analysis. Modified versions of Varimax and Orthoblique transformations (e.g., see [17] and [18]) have proven generally useful. The main difference between DEDICOM rotation and standard factor analytic rotation is that DEDICOM transformations are carried out on two planes or bimensions at a time, rather than two axes or dimensions. To accomplish this, two pairs of columns from **A** are postmultiplied by a 4 by 4 transformation matrix T which is composed of four 2 by 2 blocks of the form

$$\begin{matrix} d & c \\ -c & d \end{matrix}$$

This type of constrained transformation is said to "preserve bimensional form" (see [19], pp. 10-12, 32-33). As in factor analysis, the variance is redistributed among bimensions but the fitted sum for each x_{ij} is not changed. Thus, **R** must absorb the inverse transformations; that is, if transformed $\tilde{\mathbf{A}}=\mathbf{AT}$, then transformed $\tilde{\mathbf{R}}=(\mathbf{T}^{-1})\mathbf{R}(\mathbf{T}^{-1})'$.

When **X** is skew-symmetric, so is the corresponding **R** matrix. In the initial, "unrotated" solution, the only nonzero values in **R** are adjacent to the main diagonal; thus **R** is the skew-symmetric equivalent of a diagonal matrix (what we call "skew-diagonal"). The nonzero values correspond to the singular values of **X** (i.e., $d_{11}=r_{12}=-r_{21}$, see Appendix B). After orthogonal planewise transformations, the matrix is still skew-diagonal, but the variance is redistributed among the planes, and so the sizes of r_{12}, r_{34}, etc. are altered. After oblique planar transformations, however, additional cells of **R** are nonzero, and are organized into 2 by 2 blocks as described above. These blocks describe the strength of cross-dominances between elements in one bimensional plane and those in another (i.e., they provide an index of "obliqueness" of the bimensional planes). For preference data, this "obliqueness" indicates the extent to which preference relationships hold across bimensions. (Space limitations prevent us from discussing the algebra of bimension "rotation", or the associated changes in **R** and their interpretation; for details, see [19].)

2.3. Interpretation of Solutions

A bimension can be represented and interpreted geometrically. Each successive pair of DEDICOM axes (i.e., pair of columns from **A**) defines a plane. The stimulus coordinates on those axes can be used to plot a two-dimensional configuration of points which gives the locations of the stimuli in (their projections onto) each bimension. Gower [9] gives a geometric method by which one can interpret the resulting stimulus configuration in terms of dominance (here, preference) relationships. He points out that the dominance of stimulus y over z is proportional to the (directed) area of the triangle whose vertices are y, z and the origin. If the path from the origin to y, then to z, and back to the origin corresponds to clockwise rotation, the area--and associated dominance of y over z--is

taken as positive; if counterclockwise, negative. In this way a bimension's contribution to the dominance relationship between any two stimuli can be presented visually. We call such plots of bimensions, with the associated geometric interpretation, "Gower diagrams". In the two examples presented below, we restrict ourselves to interpreting the **A** loading matrix and associated Gower diagrams, but it is also possible, and enlightening, to look at **R**.

3. APPLICATIONS

3.1. Food Preferences

Our first example involves the analysis of an 8 by 8 set of food preferences obtained by the method of "graded paired comparisons". One person rated how much each of eight foods (four entrees and four desserts) was preferred over each of the others, using a scale ranging from -12 to +12, with 0 meaning "no preference" (Table 1). The data were then skew-symmetrized (new $\mathbf{X}=.05(\mathbf{X}-\mathbf{X}')$), and several bimensions extracted.

TABLE 1
Food preference ratings for one subject*

	1	2	3	4	5	6	7	8
1. Beef Burgundy	0	-4	-2	+4	+3	+3	+4	+5
2. Steak	+1	0	+3	+6	+1	+1	+5	+8
3. Pork Chops	0	-2	0	+4	+1	+1	+3	+5
4. Hamburger	-4	-6	-4	0	0	0	+1	+4
5. Chocolate Pie	+1	-3	0	+4	0	+1	+4	+8
6. Lemon Pie	0	-2	+1	+4	-1	0	+4	+8
7. Berry Pie	-4	-6	-4	-3	-6	-5	0	+4
8. Peach Pie	-7	-9	-7	-5	-10	-8	-6	0

*Each rating is amount row food is preferred over column food.

Unrotated solution. Gower diagrams of the first "unrotated" bimension are depicted in Figure 1. To illustrate how such a plot can be interpreted in terms of areas of triangles, we have drawn and shaded two such triangles in Figure 1a. Here (and in the other figures) the most preferred things are placed at the top of the plot so that the direction of decreasing preference corresponds to clockwise rotation about the origin. Thus, the area of the upper triangle represents the amount that Steak is preferred over Beef Burgundy Stew, and the area of the lower triangle the amount that Beef Burgundy Stew is preferred over Hamburger. Note that these two areas approximately add up to the area of the larger triangle that would be formed by connecting the origin, Steak and Hamburger. This illustrates an additive preference hierarchy, that is, a hierarchy where pref(a over b) + pref(b over c) = pref(a over c). (Collinear points represent additive hierarchies because they define a set of triangles sharing a common base --the line-- and a common vertex --the origin-- thus insuring the additivity [9]).

The first unrotated bimension provides an overall summary of the food preference relationships. The four entrees fall approximately on a straight line, as do the four desserts, but the two lines are obliquely oriented

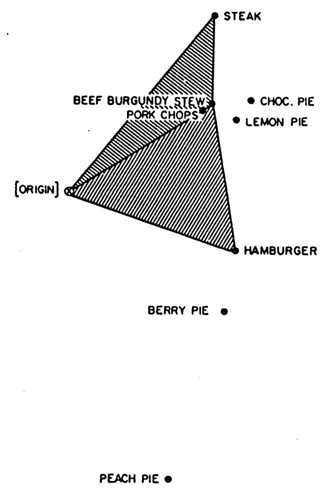

FIGURE 1a
Unrotated one-bimensional solution of food preferences data, showing geometric representation of preferences as areas of triangles.

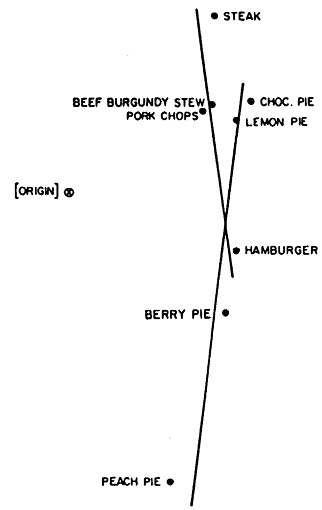

FIGURE 1b
Unrotated one-bimensional solution of food preferences data, showing linear preference hierarchies.

towards one another (Figure 1b). This suggests that the observed preference ratings may involve two separable components. To better resolve them, we obliquely rotated a two-bimensional solution, using planewise Orthoblique transformation (see [18] for a description of the Orthoblique method and [19] for the planewise transformation).

Rotated solution. The first and second rotated bimensions are shown in Figures 2a and 2b, respectively. The first bimension shows a preference hierarchy which holds mainly among desserts, while the second one shows a preference hierarchy among the entrees. Each hierarchy involves a curved rather than straight line, which indicates a modest nonadditivity in the preference relationships. This might only mean that the subject tends to underrate strong preferences relative to weak ones, or it might reflect attenuation of preference by stimulus dissimilarity, as postulated by some current theories (we say more about this later). The desserts cluster relatively close to the origin in Figure 2b and the entrees, except for Steak, do the same in Figure 2a; this suggests that the "kind of utility" represented by each bimension contributes only a small amount to the preference ratings of foods falling off the line. In terms of the subject's taste preference, it is difficult to label what the utility is that each bimension represents. Perhaps it is sufficient to say that the observed preferences can be decomposed into two simpler preference patterns, one for the desserts (and Steak), and one for the entrees (plus inter-group preferences arising from "obliqueness" of bimensions). It is also interesting to note that Steak enjoys two kinds of utility: the kind associated with other entrees, and (to a lesser degree) the special utility

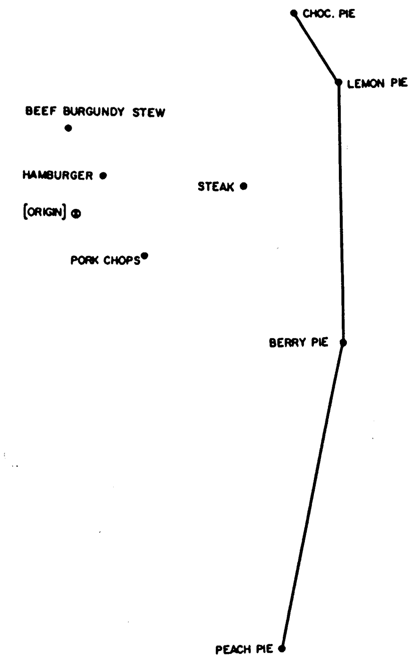

FIGURE 2a
Food preferences data, two-bimensional solution: First obliquely rotated bimension.

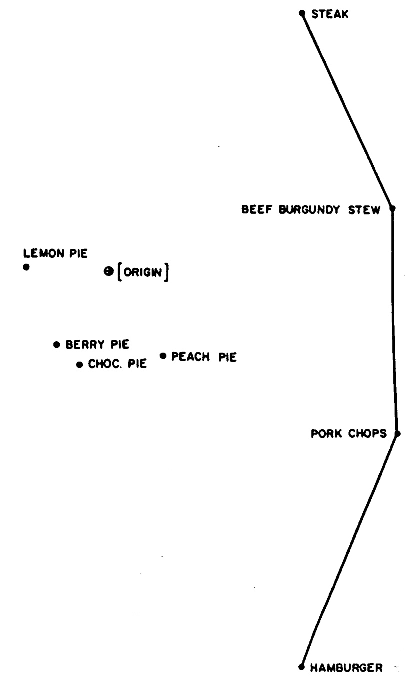

FIGURE 2b
Food preferences data, two-bimensional solution: Second obliquely rotated bimension.

associated with desserts (perhaps "enjoyment of an extravagant treat"?).

3.2. Celebrity Preferences

Our second application of DEDICOM is to a more typical kind of paired-comparison data: z-scores derived from choice proportions. The data, taken from [20], have been used by several other investigators to study similarity effects on preference strength ([4]; [7]; [21], p. 371). Thus our DEDICOM analysis can be compared to these theory-based solutions, and, in particular, to the "wandering ideal point" solution in this volume [7].

The stimuli were nine celebrities (listed in the Figure 3 legend). All possible pairs were presented, and subjects selected from each pair the celebrity with whom they would rather spend one hour, discussing a topic of their choice. The x_{ij} values for the DEDICOM analysis were obtained by converting the proportion of the sample preferring celebrity i over celebrity j to z-scores, by taking the z which cuts off the corresponding proportion of area under the normal curve.

Unrotated solution. The first unrotated plane (Figure 3) shows that Lyndon

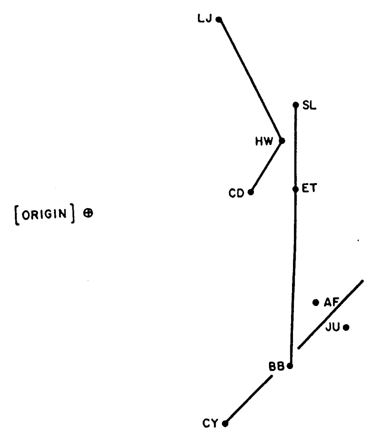

FIGURE 3
Unrotated one-bimensional solution of celebrity preferences data. LJ=Lyndon Johnson, HW=Harold Wilson, CD=Charles DeGaulle, SL=Sophia Loren, ET=Elizabeth Taylor, BB=Brigitte Bardot, AF=A. J. Foyt, JU=Johnny Unitas, CY=Carl Yastrzemski.

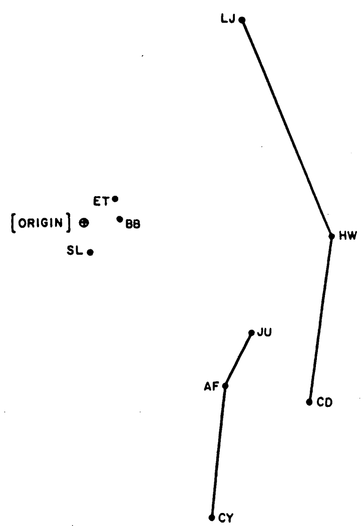

FIGURE 4a
Celebrity preferences data, two-bimensional solution: First obliquely rotated bimension.

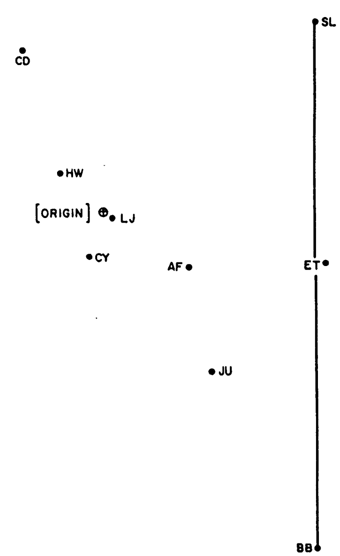

FIGURE 4b
Celebrity preferences data, two-bimensional solution: Second obliquely rotated bimension.

Johnson was by far the most preferred and Carl Yastrzemski the least preferred of the celebrities. Again there is some curvature, so that the extreme preference values are not as great as the sum of the intermediate preferences, but this may be due to several straight subsets of points, rather than a general curvature. There are indications in Figure 3 that a single overall preference hierarchy may not adequately describe these data; the politicians, movie stars, and athletes may fall on separate lines, much as the two food groups did in the prior example. (However, there seems to be something atypical about the position of Charles DeGaulle.)

Rotated solution. An oblique rotation of two bimensions was performed to see if interpretable stimulus subsets and thus identifiable "aspects of preference" could be better identified. In the rotated solution, preference relationships among actresses are shifted into the second bimension (Figure 4b), which might be labelled "glamour". Rotation also clarifies the structure of the main preference hierarchy (Figure 4a), which now shows what might be called an "importance" or "general interest" ordering for the rest of the stimuli. However, there is still some indication of two distinct groups in the first bimension, and the athletes are still nonzero in the second bimension. Thus, an oblique rotation of the three-bimensional solution was performed. (This may be an over-extraction, but it is presented for illustrative purposes). As Figure 5 shows, this solution approx-

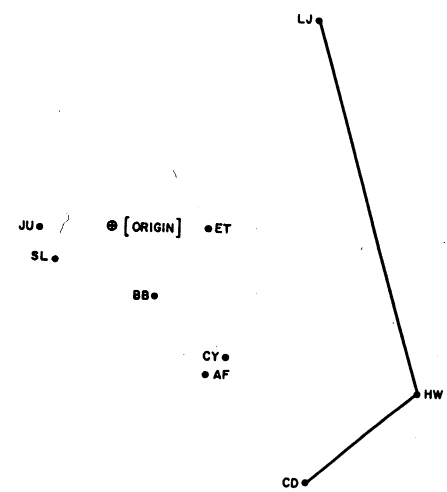

FIGURE 5a
Celebrity preferences data, three-bimensional solution: First obliquely rotated bimension.

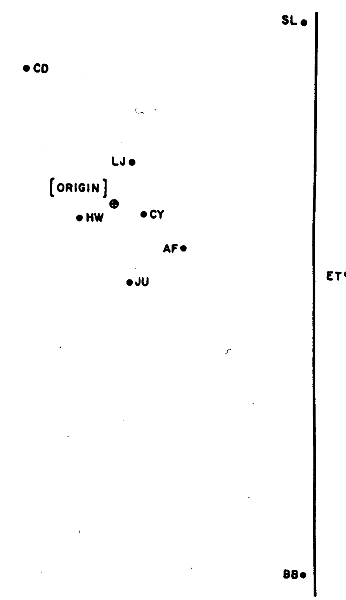

FIGURE 5b
Celebrity preferences data, three-bimensional solution: Second obliquely rotated bimension.

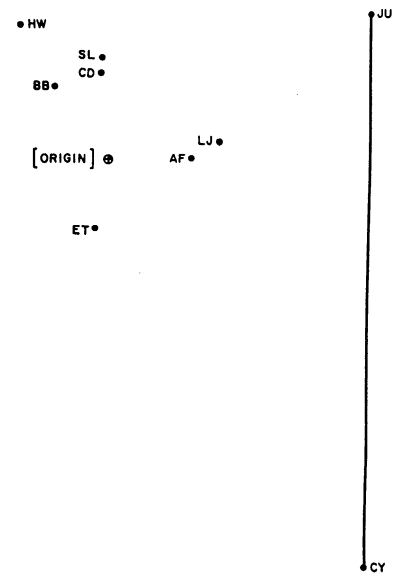

FIGURE 5c
Celebrity preferences data, three-bimensional solution: Third obliquely rotated bimension.

imately identifies a separate preference hierarchy for each of the three stimulus groups in the data. This suggests that there are three different kinds or "aspects" of utility which contribute to the overall preference judgments. One could speculate that they are the conversational interest associated with "power", "glamour" and "sports achievement".

There are still some anomalies in the patterns, however, and these might reflect subtleties of the subjects' perceptions of these stimuli. For example, Charles DeGaulle seems to take on less involvement with the first "power" bimension and more with the second "glamour" bimension than would be expected (mainly with respect to Sophia Loren). Could this have something to do with his "glamorous" historical role, or his linkage with the "glory of France"? Also, A. J. Foyt does not have the expected position on the third bimension. Of course, before taking such anomalies too seriously, it is important to establish that they are reliable (e.g., by split-half analysis, "jack-knifing" or "bootstrapping", or by comparing different samples).

4. DISCUSSION

4.1. Interpretation of DEDICOM Solutions

Information provided. The representation of preference structures in terms of component bimensions, and their depiction in Gower diagrams, reveals

several kinds of useful information. The examples above show how it can uncover (a) approximations to simple linear or additive preference scales, when appropriate; (b) nonadditive or curved preference scales, when present; (c) multiple hierarchies, if present, and their mutual obliqueness; (d) scale values for stimuli, or relative distances between stimuli on each scale or preference hierarchy; and (e) increments in variance contributed by each additional hierarchy (discussed below). It can also provide information on systematic violations of moderate or weak stochastic transitivity, and can represent circular or intransitive preferences (not illustrated here, but demonstrated in other applications).

With the food preference data, for example, a single overall scale of desirability or utility (Figure 1) captured 97% of the variance, and so a one-scale approach might be useful for most purposes. Since the points were roughly collinear, scale values for these stimuli could be approximated by taking their projections onto a single best-fitting line. Only an additional 2.7% of the variance could be fit by taking into account the split between the desserts and entrees (Figures 2a, 2b), but the discovery that this distinction affects preferences might be theoretically useful (and might have practical value in applications such as marketing). Moreover, the two-bimensional solution may provide improved scale values for the stimuli, since the locations of the entrees are no longer directly constrained by relationships to the desserts. (The entrees in Figure 2b lie on a curved line, but one way to obtain utility scale values that reflect relative preferences among them is to take their projections onto the least-squares best-fitting straight line passing near them.) Preferences between the groups are accounted for by obliqueness of the bimensions. The resulting cross-group preference strengths can be computed using the **R** matrix.

Similar results were obtained in the celebrities example, where the variance accounted for by one- through three-bimensional solutions was 96%, 98.8% and 99.9%, respectively. Again the groupings appeared meaningful, and the small size of the increments in fit after the first bimension can be attributed to the nonorthogonality of the planes.

Evaluating fit values. These fit values are quite high, but we are fitting a large number of parameters in proportion to the number of data points. (Indeed, both example data sets were fit perfectly at four bimensions.) Consequently, we should ask whether the fits of 96% and 98.7% are really higher than one should expect by chance, that is, higher than would occur if there were no structure in the data. As yet, no distributional theory allows us to analytically answer this question, and so we must use more empirical methods.

We have adopted a permutation test approach to evaluating fit values [15]. The directions of asymmetry are scrambled by exchanging the x_{ij} and x_{ji} values for a randomly determined half of the pairs in the matrix **X**. The scrambled data are then analyzed with DEDICOM and the resulting fit values obtained. The process is performed for 19 different permutations of the data. The original (unpermuted) **X** provides the 20th fit value, and all 20 are then ranked. Under the null hypothesis, all 20 datasets are equivalent and all rankings are equally likely, and so the probability that the unpermuted data would have the highest fit is .05. If it does, the hypothesis of no structure can be rejected at the .05 level.

The results of applying this test to the foods and celebrities data sets are shown in Table 2. Clearly, the fit is better than would be expected by chance at the .05 level of probability. Nonetheless, with real applications it would be preferable to have a larger data set, so as to increase the ratio of data points to model parameters. (Unfortunately, this type of test does not directly tell us how many dimensions are needed. We note, however, that the three-bimensional solutions provide uncomfortably high fit to random data.)

TABLE 2
DEDICOM Fit Values: Original and Randomly Permuted Datasets

	Food Preferences Data Bimensions Extracted			Celebrity Preferences Data Bimensions Extracted		
	1	2	3	1	2	3
Original Data						
Obtained R-SQ	.9711	.9985	.9999	.9603	.9879	.9996
Randomized Data						
Mean R-SQ	.7487	.9419	.9980	.6183	.8782	.9784
S.D.	.0560	.0256	.0026	.0844	.0626	.0221
Highest R-SQ	.8987	.9858	1.0000	.8176	.9580	.9995
Lowest R-SQ	.6420	.8838	.9888	.4995	.7388	.9250

4.2. Psychological Implications of DEDICOM Structure

The mathematical structure of a DEDICOM representation has at least two features which distinguish it from other models of paired comparison preference data: (a) the preference structure is decomposed into simple rank-2 component patterns; and (b) intransitive and weakly transitive relationships can be represented. We will now discuss what useful psychological interpretation can be given to the structures that these features permit.

Additive components as reflecting incommensurable utilities. When DEDICOM analysis reveals more than one systematic bimension, the pairwise preference relationships in the data must be too complex to be approximated by a single rank-2 preference pattern. As we have said, each basic component pattern of preferences is represented by a separate ("rotated") bimension and is generated primarily by preference relations among that subset of the stimuli that have high loadings on the associated bimension. Between subsets, preference relations are either not generated (if bimensions are orthogonal) or are attenuated (if bimensions are oblique). How might such multi-component preference structures arise? One possible psychological interpretation is in terms of incommensurable (or only partly commensurable) aspects of utility. If two stimuli are perceived to possess different kinds of utility, these utilities may be incommensurable (or partly so) and comparisons between them will be difficult. As a result, judged preferences will be attenuated. On the other hand, if two stimuli have the same kind of utility, but different amounts of it, then they are commensurable with respect to that aspect. Comparisons between them are easy and so judged preferences will not be attenuated. For example, subjects may easily compare two stimuli in terms of aesthetic value, but may find it difficult to say which they prefer if two properties are placed in conflict (e.g., if one stimulus is very aesthetic but inconvenient and the other is highly convenient but not at all aesthetic.) Under this interpretation, then, each utility dimension would generate a distinct pattern of pairwise preferences (e.g., one pattern for "aesthetic" preferences, the other for "convenience" preferences), and each such pattern would give rise to a separate bimension in the DEDICOM analysis.

An alternative: stimulus clusters. The emergence of several bimensions need not imply incommensurable dimensions of preference. An alternative interpretation is possible, one which postulates a general attenuation of preference by dissimilarity (as in the "wandering vector" and related theoretical models to be discussed later). This general attenuation could appear to operate more selectively--between, but not within certain groups of stimuli--if the stimuli happen to be arranged in tight clusters in the "similarity space." In this case, the overall preference structure would be the sum of several separate patterns of preference simply because judgments of preference between members of different clusters would be strongly attenuated relative to those within clusters; each cluster would appear as one bimension in the DEDICOM solution.

Observable differences between predictions. It would seem, then, that modulation of preferences by either dissimilarity or incommensurability gives rise to a similar DEDICOM representation. Nonetheless, the effects of the two processes are distinguishable, because incommensurability theory and dissimilarity theory make different predictions about the expected configuration of points within a bimension.

Incommensurability theory implies that attenuation of preferences occurs when dissimilarity is one of kind, but not when it is simply one of degree. Now note that only differences of degree are found within a given bimension, since the various locations of points reflect (commensurable) differences in the amount of whatever kind of utility the bimension represents. Consequently, there should be no differential attenuation of preferences within a bimension; if we adopt the usual presumption that the underlying utility is additive, the points loading strongly on the bimension should lie along a straight line. In contrast, the general dissimilarity theory predicts attenuation whenever stimuli differ in any respect. Thus even within a bimension, the preferences should be underestimated more for points falling further apart on the bimension (points that are more dissimilar) than for points lying closer together. Consequently, the set of strongly loading points should be "bent" into a curve that is concave toward the origin.

The two bimensions for the food preference example show this type of curvature, and so one might say that these data provide support for a theory of general similarity effects. (This example demonstrates how the descriptive information provided by a DEDICOM analysis can complement a "wandering vector" or related theory-based analysis.) However, there is another possible explanation. The food preference data were collected using a rating scale, and the subject may simply have avoided the endpoints of the scale; strong preferences might be underrated because of this response style.

The celebrities preference values were obtained without the use of rating scales, however. Here the evidence for curvature within bimensions is much more ambiguous. In particular, the movie stars seem to lie on an almost straight line. This suggests at least partial support for an incommensurability between the utilities resulting from interest in power and interest in glamour.

It is possible, of course, that general dissimilarity effects and specific incommensurability effects could each be found, depending on the type of stimuli. For example, data presented by Sjoberg [22, Table 5] might be interpreted as indicating both a general dissimilarity effect which attenuates preferences within dimensions, and attenuation due to partial incommensurability across dimensions.

"Factorial" models. One qualification should be noted here. Strictly speaking, the more general "factorial" models of Takane [4] and Heiser and DeLeeuw [3] need not imply a curved within-bimension preference hierarchy. Even though these models postulate the same attenuation of preference by dissimilarity as the wandering vector or wandering ideal point models, they do not imply that stimuli showing a greater difference in utility should also be more dissimilar. This is because they do not require that the similarity space contain a direction, or set of distances, onto which one can map the utilities. Instead, the similarity space is simply determined by the pattern of inferred preference attenuations which best account for the deviations from simple additivity. These "factorial" models would fit data containing linear, additive preference relationships by specifying that all stimuli are equally dissimilar to one another (lie on vertices of a simplex), and so all preferences are attenuated equally.

Although these more general dissimilarity models can account for linear, additive preference hierarchies, they do so by implying relationships that might be psychologically questionable. In particular, they have to allow stimuli with quite different values on a utility dimension to be just as similar as those with the same value (even in the absence of any "counter-balancing" differences on other dimensions). In DEDICOM terms, stimuli which were widely separated within a bimension could be just as similar as adjacent stimuli within a bimension.

Generality of DEDICOM representation. The second feature that distinguishes DEDICOM from the other models is its greater generality. It can represent data that obey only weak stochastic transitivity, and it can even handle intransitive relationships (discussed in more detail later). For example, DEDICOM can easily represent circular dominance relationships as points arranged in a circular pattern around the origin. While this generality can be quite important for other kinds of data (e.g., [23]), it is less clear how it applies to preferences. Nonetheless, circularities have been observed (e.g., [22]) and so DEDICOM's generality might be useful, at the very least, to detect and display such puzzling phenomena when they arise. While some authors have proposed intransitive utility theories (e.g., [24]) and others have offered psychological explanations for moderate intransitivity (e.g., [25]), this area of research is beyond the scope of this paper. Consequently, we will not speculate here on what psychological theories might be compatible with such preference structures. We simply note that DEDICOM will reveal such patterns, should they be present in the data.

4.3. Comparison of Theoretical and Data-Analytic Approaches

When the theory is appropriate. The descriptive information provided by a data-analytic approach can be viewed as complementary to that provided by a theory-based approach. If the theory is appropriate, the theory-based analysis gives a more illuminating representation of the underlying process that generated the data, including estimates of underlying utility values for the stimuli being fitted, and stimulus similarities that might be

useful for, say, marketing purposes. On the other hand, a data-analytic approach like DEDICOM gives a more detailed description of the preference structure that actually exists in the data. Some of these descriptive details can also be valuable for marketing and forecasting purposes.

A DEDICOM analysis reveals both regularities and anomalies in the preference structure, and both can be used by the applied researcher. For example, the occurrence of any stimuli in anomalous or unexpected positions (as Charles DeGaulle in our analysis) signals unusual preference patterns involving those stimuli. The researcher might thus be alerted to important characteristics of the stimuli requiring further study. On the other hand, regular patterns, as indicated by orderly stimulus arrangements on one or more bimensions, may suggest distinct aspects of preference that need to be considered when designing or marketing new products.

When a DEDICOM bimension reveals linear or simple curvilinear preference hierarchies (e.g., Figures 2, 4b), one interpretation would be in terms of stimulus differences in the amount of some specific attribute. Product designers might use such information to estimate how varying that attribute in a new product would change the preference for the product. On the other hand, suppose two groups of points fall on differently oriented lines within a given bimension, or on different bimensions. Then the researcher would have reason to believe that more complex relationships were involved, and a product change that would strongly alter the product's preference relationships with respect to one group would probably not produce such large changes with respect to the other group.

When the theory is questionable. When it is unclear which psychological theory is appropriate, or when the data contain theoretically troublesome preference relationships, DEDICOM analysis might be useful precisely because its method of representation is much more general than those based on specific models. Thus, it can deal with a much wider range of preference patterns and provide a useful description of the data structure, even in the absence of a theory explaining why the structure would have a given form. Good descriptive information about that form is valuable to those involved in marketing, etc., whether or not the "wandering vector" or some other psychological model is appropriate. In addition, by studying the structure obtained, one might gain insights that would suggest a general theory that could account for the observed patterns.

We can compare the generality of various models by considering the different restrictions they would place on the arrangement of points in a DEDICOM representation. Figure 6 shows the range of relationships that can be represented by several psychological models of preference, compared to what DEDICOM can represent. Points in the figure are arranged in a circular arc around the origin of a DEDICOM bimension. A highly restricted utility model such as Thurstone Case V [26] would require that all points fall on the straight line, since it implies that the fitted parts of the data obey a linear additive preference hierarchy. "Strong" utility models such as Bradley-Terry-Luce ([27]; [28]) would not require linear additivity, but would require that the relations obey strong stochastic transitivity (i.e., when a is preferred over b and c, and b is preferred over c, pref(a over c) is greater than or equal to the larger of pref(a over b) and pref(b over c)). In the circular bimension shown in Figure 6, this would correspond to the relationships among points within a 90 degree arc, such as the points to the right of the vertical line.

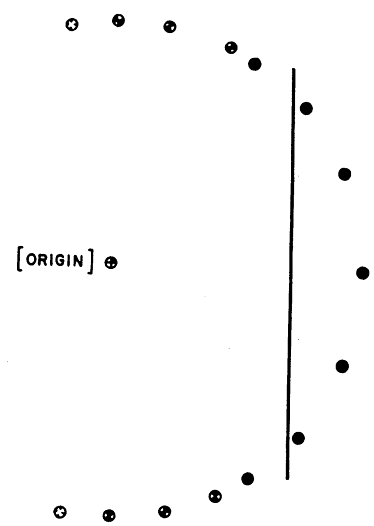

FIGURE 6
Diagrammatic representation of relationships that conform to various kinds of transitivity.

Weaker restrictions are also depicted in Figure 6. Solid points in the figure lie on a 120 degree arc, where moderate stochastic transitivity holds (i.e., pref(a over c) is greater than or equal to the smaller of pref(a over b) and pref(b over c)). Within 180 degrees of arc, the relationships obey weak stochastic transitivity (i.e., a preferred over b, and b over c, implies a is preferred over c), and beyond 180 degrees, the relationships begin to show intransitivities (circular triads). Only relationships that obey strong or moderate stochastic transitivity (i.e., involving the solid points in the figure) can be represented by current theory-based methods such as "wandering vector" or "factorial" models. In contrast, DEDICOM can represent all the different kinds of relationships shown in Figure 6, and even completely intransitive ones.

4.4. Other Applications

The relevance of the methods used above, and their possible application to telecommunications problems, should be apparent. Potential customers could be asked to rate pairwise preferences between possible options for telecommunications service, such as different bandwidths, toll options, etc. Or, different hardware configurations could be compared. The analysis would then be directed at uncovering the underlying dimensions of preference and inferring those aspects or features of the products which determine relative preference, as well as quantifying the relative positions of the product on each preference dimension and the relative contributions of each dimension to total overall preference. Careful construction of the response scales (e.g., by asking the subjects to state how much they would be willing to pay to have one option over another) might make it possible

to calibrate the preferences in terms of actual dollar values. Then the areas of the triangles in each bimension would represent the difference in price needed to produce indifference between alternatives, when the contributions of only that aspect of preference is extracted from the total.

The general DEDICOM model can also be applied to a matrix in which x_{ij} represents the amount of telephone traffic between points i and j. If the skew-symmetric part of the data is extracted, so that x_{ij} represents the imbalance of telephone traffic between different points, the bimensional representation used above would apply. To explore this area of application, we performed a DEDICOM analysis of the skew-symmetric part of a data set giving the telephone traffic between 16 regions in the U.S.A. One result of the analysis is shown in Figure 7. As before, clockwise rotation represents dominance. For example, the size of the triangle between the

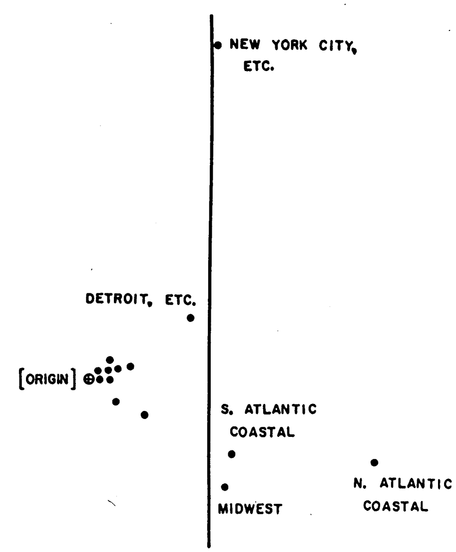

FIGURE 7
Skew-symmetric part of telephone traffic data: First bimension of a DEDICOM two-bimensional solution.

origin, "New York City, etc." (including the New York City, New Jersey and Washington, D. C. areas) and "Detroit, etc." (including the Detroit, Chicago and Cleveland-Pittsburgh metropolitan areas) represents the imbalance in calls between these two regions (more calls from NY to Detroit).

Overall, this bimension shows a linear imbalance hierarchy that seems to order the regions according to urbanization and population. The main exception to the linear pattern is the North Atlantic Coastal region (which includes Maine, Vermont, New Hampshire, part of New York state, part of

Pennsylvania, Virginia and southern Florida). Falling outside the hierarchy, it reflects an extra large imbalance with New York City, etc. This anomaly might be due in part to the proximity of most of this region to New York City, etc. In general, this bimension reveals a very orderly (linear and additive) tendency for the difference in urbanization level of two regions to be reflected in the imbalance of telephone traffic between them. We see a pattern originating in the eastern USA for a relatively more urbanized region to generate more calls to a less urbanized region than vice versa. The second bimension (not shown here) describes patterns of imbalance between other areas; more detailed analyses may suggest patterns due to factors other than urbanization.

4.5. Conclusion

We have presented an exploratory or "data-analytic" approach to studying the structure underlying a paired comparison preference matrix, based on the DEDICOM generalization of multidimensional scaling or factor analysis. It can reveal salient dimensions useful for product design and marketing, and can also highlight peculiarities of preferences involving particular stimuli deserving of further study. DEDICOM analysis is viewed as complementary to the more theoretically based analysis methods such as the "wandering vector", which fit specific psychological models to the preferences. Both approaches are useful for gaining a deeper understanding of the detailed structure underlying a pattern of pairwise preferences among several stimuli.

5. APPENDIX A

5.1. The General DEDICOM Model

Consider an n by n data matrix **X**, with the rows and columns representing the same set of entities. The entry x_{ij} represents the directed relationship (e.g., amount of telephone traffic, brand switching, etc.) from entity i to entity j. Often, such directed relationships are asymmetric; that is, $x_{ij} \neq x_{ji}$. DEDICOM represents the underlying structure in X in a way that sheds light on systematic asymmetric as well as symmetric aspects.

The (single domain, two-way) DEDICOM representation of X has the form $\mathbf{X} = \mathbf{ARA}' + \mathbf{E}$, where **A** is an n by s matrix giving "loadings" of the n stimuli on s dimensions, **R** is an s by s matrix of relationships among the dimensions, and E is a matrix of residual error terms. Asymmetries in **R** reflect systematic asymmetries in X, and R can be interpreted in the same terms as **X**; it gives a "miniature" version of **X**, but at a higher level of description. For example, if X were a matrix describing how many persons switched from the row brand to the column brand in some period of time, then the columns of **A** would give types or aspects of products, and **R** would give a description of switching between these different types or aspects.

As in the skew-symmetric case, solutions are "rotated" to maximize interpretability. However, in the general asymmetric case, two dimensions are rotated at a time, rather than two planes at a time. The interpretation of the model for general asymmetric **X** is discussed in more detail elsewhere ([14]; [15]).

6. APPENDIX B

6.1. Least-squares Fitting of DEDICOM in the Skew-Symmetric Case

Let the singular value decomposition (SVD) of **X** be written

$$(2) \qquad \mathbf{X} = \mathbf{\overset{*}{U}\overset{*}{D}\overset{*}{V}}' \quad ,$$

with $\overset{*}{\mathbf{D}}$ diagonal and $\overset{*}{\mathbf{U}}$, $\overset{*}{\mathbf{V}}$ orthogonal. The s-dimensional (s/2 bimensional) "unrotated" DEDICOM representation can be obtained from (2) by defining **A=U** and **R=DP**, where U is the orthonormal section composed of the first s columns of $\overset{*}{\mathbf{U}}$, **D** is composed of the first s rows and columns of $\overset{*}{\mathbf{D}}$, and **P** is an s by s permutation (and sign changing) matrix, of the form

$$\begin{matrix} 0 & 1 & 0 & 0 & \cdots \\ -1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ 0 & 0 & -1 & 0 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{matrix}$$

In the SVD of any skew-symmetric matrix, $\mathbf{U}' = \mathbf{P}'\mathbf{V}'$. Consequently, we can obtain the DEDICOM representation from the SVD by taking advantage of the fact that $(\mathbf{PP}' = \mathbf{I})$ and inserting it into the rank-s SVD approximation of **X** as follows:

$$\mathbf{X} = \mathbf{UD(PP')V'} = \mathbf{U(DP)(P'V')} = \mathbf{URU'} = \mathbf{ARA'}$$

After this transformation, the **R** matrix is skew-symmetric, with the singular values adjacent to the main diagonal (e.g., $d_{11} = r_{12} = -r_{21}$) and all other values zero. This DEDICOM solution has orthogonal bimensions, with maximum variance accounted for by the first bimension, and maximum portions of residual variance accounted for by each successive bimension. This "principal planes" solution is analogous to the unrotated rank-s principal components approximation of a symmetric positive definite or semidefinite matrix [19, p. 13]. As noted earlier, it is usually linearly transformed into an alternative form before interpretation.

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