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An Application of PARAFAC to a Small Sample Problem, Demonstrating Preprocessing, Orthogonality Constraints, and Split-Half Diagnostic Techniques[1]

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In this appendix, we present in detail the procedure involved in a single application of PARAFAC three-way factor analysis to real data. The data are from a pilot marketing study and consist of the ratings of 25 stimuli (names of automobiles and celebrities) made by each of 34 raters, using a set of 39 bipolar rating scales. The objective of the analysis is to determine the connotative and semantic dimensions describing the celebrities and automobiles in order to decide which celebrity should be chosen as spokesman for a given automobile.

This application is intended to provide a demonstration/explanation of what PARAFAC is and what it does, for those to whom "an example is worth a thousand equations." It is also intended to serve as a guide for those who perform or are about to perform their own analysis of a three-way data set. The data are typical, since stimulus ratings by each of several subjects on each of several ratings scales is probably one of the most common kinds of three-way social sciences data. Since it is a small data set, it highlights the need for good methodology to enable maximum information recovery without misinterpretation of random error as meaningful patterns. The analysis problems that come up are the ones frequently encountered in analyzing such data; they are also typical of many other kinds of three-way data. The techniques used to preprocess the data, to test dimensionality, to determine stability of the solution, and so on are the basic ones needed in all careful three-way analyses. Thus, this article could be viewed as an illustrative companion to the general discussion of diagnostics provided by Harshman (see appendix A).

Factor analysis could never be properly conducted in a "one-shot" run through the computer—unfortunately, too many social scientists have used it that way. Its proper use has always required careful planning, followed by a series of analyses. In the analysis stage, study of the results of a given analysis is followed by reanalysis, extracting a different number of factors, using a different rotation procedure, and so forth. Based on what is learned from each solution, a new analysis is performed and the process is repeated until the optimal solution is obtained. The need for such a careful, multistage analysis *process* is even greater with PARAFAC three-way factor analysis, and this article provides an example of how it is done.

This application demonstrates: (a) three-way preprocessing and its effects on the solution; (b) the method of split-half cross-validation and how it is used to test the stability and reliability of each solution; (c) how comparison of split-half results for different numbers of factors is used to establish the maximum number of dimensions; (d) how application of orthogonality constraints to one loading matrix is used to overcome "degenerate" solutions and permit recovery of additional meaningful dimensions; and (e) the process of interpretation of a three-mode solution—how the loadings from all three modes (scales, stimuli, and subjects) can be used to strengthen insight into the nature of each dimension and how comparison of interpretations across several dimensionalities helps to refine judgments about the "proper" dimensionality and the meaning of the factors that are obtained.

While the use of diagnostic procedures is always important (see appendix A), it becomes particularly important in this type of application because of the small subject sample that was employed (only 34 in all, 17 in each split-half). The results demonstrate how proper use of split-half validation can be used to determine the number of dimensions that can in fact be reliably extracted from such small samples and how stable the extracted dimensions are. The fact that at least three (and probably four or five) dimensions could be reliably recovered from split-half samples as small as 17 shows just how encouragingly robust and powerful these three-way methods are.

THE PROBLEM

Motivation for the Analysis

The example is drawn from a marketing application in which the motivating question is: How can one select an appropriate commercial spokesman for a given brand or product? By applying three-way factor analysis to semantic differential rating scale data on products (in this case, automobiles) and potential spokesmen, we hope to discover the underlying connotative overtones of the spokesmen and products, enabling us to display both in a common multidimensional *semantic space*. By using such a space, we will be able to make better informed judgments about how the overtones of a particular spokesman might reinforce or interfere with the desired impression for a particular product. This type of marketing application is a special case of a methodology called *connotative congruence analysis,* which is discussed in more detail in the longer manuscript from which this appendix was taken (Harshman and De Sarbo 1981).

The Data

The data consists of ratings of each stimulus word on 39 bipolar seven-point scales. The stimuli used are given in Table C–1, and the concepts labeling the poles of the rating scales are given in Table C–2. The approach is based on the *semantic differential* technique (Osgood 1962; Osgood, Suci, and Tannenbaum 1957). At the top of each page of the subject's test booklet was the name of a celebrity or car make; on the twenty-fifth page, the subjects rated themselves on the same 39 scales. Below the stimulus name were the 39 bipolar rating scales. The subject would place a mark somewhere along each bipolar scale, indicating how strongly he thought the stimulus named at the top of the page was related to the adjective at one end or the other of the

TABLE C-1. Stimulus List

Twelve Celebrities Tested (Aided Recall):		
1.	Bob Hope	(Comedian)
2.	John Wayne	(Actor)
3.	Muhammed Ali	(Boxer)
4.	Farrah Fawcett	(Model)
5.	Ralph Nader	(Consumer Rights)
6.	Orson Welles	(Drama)
7.	Sammy Davis, Jr.	(Singer)
8.	Arnold Palmer	(Golfer)
9.	Jerry Lewis	(Comedian)
10.	John Travolta	(Actor)
11.	Barbara Walters	(Newscaster)
12.	Mary Tyler Moore	(Actress)
Twelve car makes tested:		
13.	Ford	
14.	Buick	
15.	Chevrolet	
16.	Cadillac	
17.	Oldsmobile	
18.	American Motors	
19.	Chrysler	
20.	Dodge	
21.	Plymouth	
22.	Lincoln	
23.	Pontiac	
24.	Mercury	
25.	Self	

TABLE C-2. Bipolar Adjectives for the 39 Semantic Differential Scales

	Adjective pair			Adjective pair	
1.	Pleasant	--- Unpleasant	21.	Expert	--- Novice
2.	Strong	--- Weak	22.	Masculine	--- Feminine
3.	Formal	--- Informal	23.	Slow	--- Fast
4.	Dynamic	--- Static	24.	Superior	--- Inferior
5.	Usual	--- Unusual	25.	Ugly	--- Beautiful
6.	Colorless	--- Colorful	26.	Simple	--- Complex
7.	Leading	--- Following	27.	Trustful	--- Distrustful
8.	Plain	--- Ornate	28.	Austere	--- Lush
9.	Sophisticated	--- Naive	29.	Smooth	--- Rough
10.	Liked	--- Disliked	30.	Public	--- Private
11.	Disreputable	--- Reputable	31.	Obscure	--- Famous
12.	Superficial	--- Profound	32.	Old	--- New
13.	Mature	--- Youthful	33.	Orthodox	--- Heretical
14.	Rational	--- Intuitive	34.	Graceful	--- Awkward
15.	Familiar	--- Strange	35.	Efficient	--- Inefficient
16.	Positive	--- Negative	36.	Light	--- Heavy
17.	Careless	--- Careful	37.	Interesting	--- Boring
18.	Aggressive	--- Defensive	38.	Large	--- Small
19.	Hard	--- Soft	39.	Attractive	--- Unattractive
20.	Active	--- Passive			

scale. For example, one stimulus name was "John Wayne" and one of the rating scales was "Light 1 2 3 4 5 6 7 Heavy." If the subject thought John Wayne was "Heavy" in some metaphorical sense, he would circle a number on the right-hand side of the scale. The "heavier" he thought Wayne to be, the further to the right his circle would be.

The 39 scales were selected to tap the basic semantic differential dimensions of Evaluation, Activity, and Potency and also to get at qualities related to previously published theories of "source credibility" (Kelman and Howland 1953; McGuire 1969): Expertness, Attractiveness, Trustworthiness, and Likability. We also included other aspects thought relevant to connotative congruence of these stimuli (see Mowen 1980). In addition, each rater also answered 30 questions regarding attitudes and driving styles. For example, some of the items were: "I usually look for the lowest possible prices when I shop" and "I admit I try to keep up with the Joneses." These were collected to shed possible light on the market segments that particular subjects might represent and to aid in the interpretation of the person loadings that would eventually be used to help interpret the dimensions.

The data input to the PARAFAC analysis consisted of a 39 × 25 × 34 array. Each row of the input data corresponded to the ratings of the 25 stimulus items on one of the 39 scales; each set of 39 rows corresponded to the ratings provided by one subject.

DATA ANALYSIS

Preprocessing

To demonstrate the importance of data preprocessing, we first present the results of a three-dimensional analysis of the raw data (Table C–3). We see here a more or less uninterpretable solution. The first dimension is probably adjusting for the different constant offsets of the various rating scales from a true zero origin. Note that the mean value in Mode *C* (the mode chosen to reflect the scale of the data in this study) is around 3.5, which is the center of the 7-point rating scale. The different loadings in Mode *B* range from close to zero to close to 2.0. This would reflect differences in the additive constant for the different scales. The pattern of variations in this mode suggests that the zero-point was taken to be that point on the scales corresponding to high positive evaluation of the stimuli. Dimensions 2 and 3 show the classic pattern of degeneracy discussed by Harshman and Lundy (chapter 6). As shown in Table C–4, the loadings are highly correlated in all three modes and the product of the three correlations—that is, Mode *A* × Mode *B* × Mode *C* correlation—is negative.

Before preprocessing, the data is liable to contain unwanted constants and two-way interactions that interfere with the ability to define axes uniquely. For one thing, PARAFAC expects ratio-scale data and the raw data is interval-scale at best. Additional reasons why unpreprocessed data may not provide meaningful solutions are discussed in Harshman and Lundy (chapter 6). To overcome this degeneracy, the data were centered on Modes *A* and *B* (stimuli and scales) and size-standardized on Modes *B* and *C* (scales and subjects). This double-centering removes the overall additive constant and all one-way "main effects," as well as subject-stimuli and subject-rating scale interactions, since these are constant across Mode *B* and Mode *A*, respectively. The data were size-standardized on Mode *B* (rating scales) because it was thought that some scales might show much less variance than others simply as an artifact of the choice of overly extreme labels or because of "ceiling effects" (such as all celebrities being rated as "famous"). Size standardization permitted all the rating scales to be approximately equally weighted in the analysis and provided comparability of loadings across levels of Mode *B*. The subject mode was also size-standardized to ensure that all subjects contributed equally to the solution and to remove differences in the size of responses that might arise from response styles such as "extreme" responding versus "moderate" responding. This combination of preprocessing options has been repeatedly found to give good results with three-way rating scale data. The preprocessing required four iterations to reach the standard convergence criterion (less than .01 deviation from requested equality of mean-squares in all modes).

Which precise combination of centering and standardization options to select is not always obvious, although the nature of the application and the data collected can render valuable insights into appropriate preprocessing options. In general, one must consider which additive constants, two-way interactions, and

TABLE C-3. PARAFAC Three-Dimensional Unconstrained Solution for Raw Unpreprocessed Data

	Mode A		
	1	2	3
1	.96	.73	.53
2	.88	1.07	1.21
3	1.06	.50	.14
4	1.06	.19	-0.89
5	.99	1.29	1.46
6	1.01	1.36	1.74
7	1.04	.45	-0.18
8	.93	.96	.88
9	1.04	.67	.08
10	1.04	.27	-0.80
11	.99	1.06	.95
12	1.02	.43	-0.13
13	.94	1.18	.86
14	1.00	1.20	1.23
15	.96	.88	.44
16	1.06	1.33	1.81
17	1.00	1.14	1.10
18	.97	.96	.06
19	1.02	1.47	1.56
20	.96	1.02	.42
21	.97	1.29	.85
22	1.05	1.37	1.92
23	1.01	.83	.39
24	1.03	.91	.56
25	1.00	.88	.73

	Mode B		
	1	2	3
1	.25	1.14	-0.97
2	.30	1.22	-1.41
3	.66	.99	-1.40
4	.11	1.31	-0.99
5	1.66	-0.90	.75
6	2.02	-1.41	1.14
7	.21	1.42	-1.45
8	1.71	-1.07	.87
9	.34	1.28	-1.43
10	.16	1.21	-1.02
11	1.74	-0.85	1.04
12	1.14	-0.49	.89
13	.72	.70	-1.23
14	.94	.39	-0.86

TABLE C-3. Continued

	Mode B		
	1	2	3
15	.20	.96	-0.84
16	.09	1.37	-1.19
17	1.46	-0.66	.99
18	.23	1.19	-1.08
19	.85	.33	-0.47
20	.09	1.24	-0.91
21	.27	1.29	-1.48
22	.68	.63	-0.89
23	1.65	-0.72	.37
24	.16	1.42	-1.42
25	1.74	-0.98	.75
26	1.35	-0.71	1.02
27	.49	.78	-0.80
28	1.58	-0.80	.66
29	.26	1.14	-0.98
30	.58	.42	-0.09
31	2.06	-1.09	.87
32	1.12	.01	-0.49
33	1.14	-0.22	-0.04
34	.14	1.37	-1.08
35	.40	.91	-0.74
36	.91	-0.09	.75
37	.27	1.30	-1.16
38	.73	.73	-1.18
39	.20	1.28	-1.03

	Mode C		
	1	2	3
1	3.71	3.08	1.54
2	3.74	3.78	2.24
3	3.94	3.20	1.98
4	3.67	3.28	1.47
5	3.76	3.30	2.02
6	3.79	3.59	2.00
7	3.86	3.66	2.01
8	3.79	3.17	1.48
9	3.79	3.69	1.84
10	3.70	2.84	1.60
11	3.81	3.93	2.16
12	3.76	4.07	2.28
13	3.82	3.59	2.15
14	3.72	3.88	2.07
15	3.82	4.11	2.27

TABLE C-3. Continued

	Mode C		
	1	2	3
16	3.77	3.22	1.80
17	3.71	3.22	1.49
18	3.82	3.31	1.56
19	3.87	4.37	2.61
20	3.47	3.36	2.10
21	3.71	3.32	1.95
22	3.98	2.51	1.53
23	3.88	3.47	2.03
24	3.62	2.80	2.12
25	3.55	2.56	1.28
26	3.70	2.71	1.45
27	3.78	2.87	1.41
28	3.79	3.37	1.70
29	3.68	3.32	1.57
30	3.67	3.84	2.05
31	3.45	1.92	1.17
32	3.64	2.33	1.16
33	3.69	2.66	1.60
34	3.80	3.38	1.56

Root-Mean-Squared Contribution for Each Factor		
1	2	3
3.744	3.328	1.835

FIT (R^2) = .337

inequalities of variance are most likely to cause problems. A certain amount of trial and error is often necessary to check alternative preprocessing schemes. (Indeed, in the example considered here, we tried an additional standardization of variances for the stimuli but decided that meaningful differences in overall concept salience were obscured and so did not use this standardization in our final analyses.) The preprocessing presented above should be considered as one of several useful possibilities.

Factor Analysis Procedures

The analysis proceeds in a stepwise fashion, moving from a set of one-dimensional analyses to the two-dimensional analyses, and so

TABLE C-4. Correlations of Factor Loadings for Analysis of Unpreprocessed Data

	Mode A		
	1	2	3
1	1.00	-0.31	-0.21
2	-0.31	1.00	.93
3	-0.21	0.93	1.00
	Mode B		
	1	2	3
1	1.00	-0.98	.89
2	-0.98	1.00	-0.95
3	.89	-0.95	1.00
	Mode C		
	1	2	3
1	1.00	.43	.35
2	.43	1.00	.83
3	.35	.83	1.00

on, until various diagnostics (to be described) indicate that too many factors have been extracted. A given step consists of using several different random starting positions to obtain several PARAFAC solutions at the given dimensionality, followed by the application of a number of comparisons and diagnostic checks to evaluate convergence, optimality, stability, and generalizability of the solutions obtained. If the diagnostics indicate that the solutions obtained are appropriate, we proceed on to the next higher dimensionality. After covering a range of dimensionalities, further comparisons of loadings and fit values across different dimensionalities provide a basis for selecting the "correct" solution(s) for final interpretation. (For an introductory survey of diagnostic procedures for three-way factor analysis, see appendix A in this volume.)

To select the preferred solution at each dimensionality, and to determine the "correct" dimensionality for final interpretation, we employed several interrelated techniques: (1) to evaluate the optimality and stability of the factors obtained at each dimensionality, we obtained and compared three independent solutions using three different random starting points for the iterative procedure;[3] (2) the fit values of optimal, converged solutions were plotted as a function of the number of factors extracted, in order to estimate the dimensionality beyond which additional dimensions would only produce small, gradual improvements in fit, attributable to fitting "noise" in the data; (3) the correlations among dimensions within each solution were examined in order to check for highly correlated dimensions, indicative of extracting too many dimensions or other such problems; (4) the reliability and generalizability of the dimensions obtained at a given dimensionality were checked by comparing results obtained from different split-halves of the data set; (5) the interpretability of the results was examined at each dimensionality; (6) the evolution of the interpretations obtained at successive dimensionalities was examined by comparing different dimensional solutions.

An initial series of PARAFAC analyses was performed without imposing constraints concerning the orthogonality or obliqueness of dimensions. After certain diagnostics suggested that orthogonality constraints might be useful, an additional series of analyses was performed with the dimensions constrained to be orthogonal in Mode *B* (that is, across rating scales). The original objective of this constrained procedure was to clarify dimensions obtained in the unconstrained analyses. It turned out, however, to reveal additional structure in the data. We describe the details of the unconstrained analyses first.

Unconstrained Analyses

The initial series of analyses was performed in one through seven dimensions. At each dimensionality, three independent random starting positions were used for the iterative procedure to check for local optimum solutions. The resulting three solutions were compared to evaluate the stability or uniqueness of the dimensions and to reduce the chance of being misled by a local optimum. The loading patterns for the obtained dimensions were correlated across the three solutions to measure their agreement (using the PARAFAC utility program CMPARE). Stable solutions were obtained in all dimensionalities between one and five, as indicated by correlations of .999 between corresponding dimensions from the three different starting positions. At six and seven dimensions, some solutions did not converge and others converged to different places. (We used the default PARAFAC criterion for convergence; that is, from one iteration to the next, no loading should change more than one-tenth of one percent of the root-mean-square average loading value on that factor in that mode.) Because various diagnostics pointed to three as the highest dimensionality that could be relied on with this data set (as will be explained below), an extended effort was not made to determine the optimal, converged form of the solutions for five- and higher-dimensional solutions.

Fit Values. At each dimensionality, several different goodness-of-fit measures were computed. The R^2 values for the 1 through 7 dimensional constrained solutions were as follows: 1D = .123; 2D = .195; 3D = .258; 4D = .293; 5D = .333; 6D =

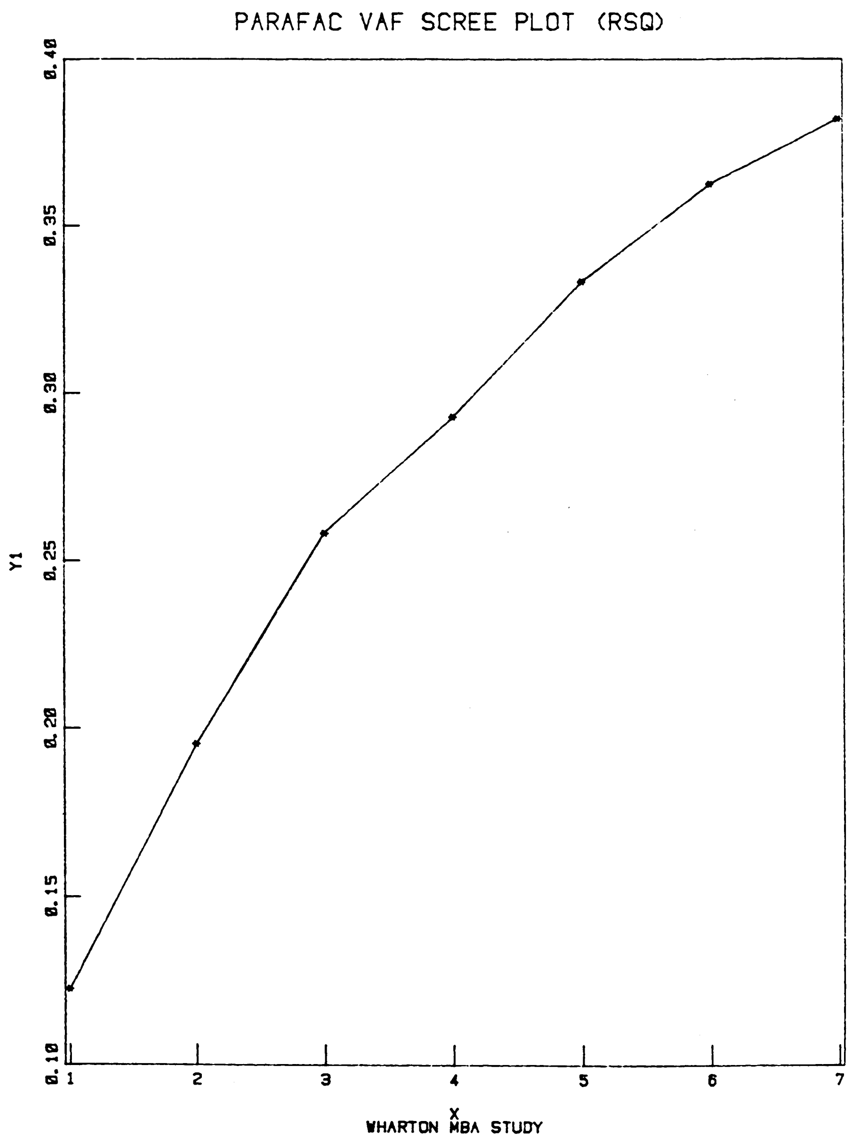
.363; 7D = .383. Examining the R^2 values, we see that by using three dimensions, PARAFAC is able to account for roughly 25% of the data variance. How good is this figure? Experience with other three-way data, plus some Monte Carlo simulations, suggests that these values are not particularly high, but they are high enough to be compatible with correct recovery of major dimensions in the data. It must be remembered that PARAFAC fits the "uncollapsed" three-way array and thus these figures represent the ability of the factors to reproduce individual ratings. They should not be compared to fit values from studies using averaged data. Yet, while PARAFAC fits the individual ratings, it fits all the raters at the same time, and so it is able to take advantage of patterns that are consistent over many raters to improve the accuracy of its estimates of the loadings for stimuli or scales. This gives PARAFAC an ability to detect patterns that would normally only become apparent in averaged (collapsed) data.

Traditionally, two-way analyses of this three-way data might typically deal with ratings averaged over the thirty-four subjects, and such averaging would reduce the error variance of the data and would thus improve the fit values of the analysis considerably. To see what the fit value would be with a more traditional analysis of this data set, we "collapsed" the 25 × 39 × 34 array into a 25 × 39 × 1 array of mean ratings by averaging over subjects. When these "collapsed" data were analyzed by PARAFAC, the dimensions obtained were, of course, not unique in axis orientation. But consistent fit values were obtained from different starting positions. In three dimensions, a fit value of R = .829 was obtained, which means that PARAFAC could account for 68.7% of the variance of the averaged data by using three factors. This higher fit value should reassure those investigators who are used to examining fits for two-way analyses of averaged data, since PARAFAC analysis of the "uncollapsed" data should provide equivalent accuracy in the recovery of the underlying structure, with the additional advantage of determining the orientation of axes uniquely.

Another way one might express the same concern about the seemingly "low" R^2 of .25 is by asking whether this reflects the ability of the model to recover structure or simply its ability to fit "noise" in the data. To provide some information on this, we took advantage of the Monte Carlo data synthesis options available in PARAFAC and constructed an array of synthetic data similar to ours but consisting entirely of random noise. In particular, we constructed a 25 × 39 × 34 array of uniformly distributed random deviates, with a constant standard deviation of 1.0. We then preprocessed this array in the same way as the real data had been preprocessed and submitted it to PARAFAC for analysis in three dimensions, from three different random starting positions. The resulting solutions were all different (presumably because there were no systematic dimensions in the data), but the solutions all had approximately the same R^2 values: .0224, .0223, and .0214. The R^2 in all cases indicated that PARAFAC could fit only about two percent of the error of an array of this size. Since it could account for 25% of the variance of our real data, it seems clear that this figure is not primarily due to fitting noise in the real data (at least if our admittedly simple Monte Carlo example is any indication).

To obtain an estimate of the number of major dimensions underlying the patterns of variation in the data, several plots of fit versus dimensionality were made. Figure C–1 shows one such plot, based on R^2, or the variance accounted for by the PARAFAC model at each dimensionality. Preliminary results of Monte Carlo studies suggest that the plots of R^2 may provide the best indication of "true" dimensionality. The increases in explained variance should be large at first, because each higher dimension-

Figure C–1. Plot of Dimensionality versus R^2 for Unconstrained Solutions



ality incorporates another major factor explaining a new systematic aspect of the variations in the data. However, when the remaining variance is primarily random error, extraction of additional dimensions should produce only a small and consistent increase in R^2 at each step. Figure C–1 does not have a striking "elbow," but might suggest that there are 3 major and 2 to 3 minor additional dimensions in these data. Plots of other measures of fit suggest slightly lower bounds, with R suggesting 3 dimensions and $\log(1 - R^2)$ suggesting that there are just two major factors. Thus, as sometimes happens, the plots of fit versus dimensionality are ambiguous, none containing striking and unequivocal evidence for a given dimensionality. Thus, we are forced to rely even more strongly on more fundamental indications of dimensionality, based on correlations both within and across split-half solutions.

Within-Solution Correlations among Dimensions. One indication that too many factors are being extracted from a given data set is the occurrence of highly correlated factors in all modes. One factor may "split up" into 2 similar versions of itself or more general mixing and overlap of factors may occur. With our data, this phenomenon occurred when we went from 3 to 4 factors. The correlations between factors within each mode are shown for the two- and three-dimensional solutions in Table C–5. The highest correlations are in Mode *A*, corresponding to similar patterns of loadings across celebrities and automobile makes. Even for this mode, however, the highest correlation in the three-dimensional solution is .41 and in the two-dimensional solution is .50. Contrast these moderate correlations with the very high correlations between dimensions one and two in the four-dimensional solution displayed in Table C–6. (This pair of dimensions corresponds to a splitting up of what was dimension two of the three-dimensional solution; dimension one of the three-dimensional solution has become dimension three, and the old

TABLE C-5. Correlations among Dimensions for Two- and Three-Dimensional Unconstrained Solutions

		Two Dimensions			Three Dimensions		
		1	2		1	2	3
Mode A	1	1.00	.50	1	1.00	-.36	.26
	2	.50	1.00	2	-.36	1.00	-.41
				3	.26	-.41	1.00
Mode B	1	1.00	-.24	1	1.00	.01	-.12
	2	-.24	1.00	2	.01	1.00	.24
				3	-.12	.24	1.00
Mode C	1	1.00	.14	1	1.00	-.11	-.14
	2	.14	1.00	2	-.11	1.00	.39
				3	-.14	.39	1.00

TABLE C-6. Correlation within Modes for the Four-Dimensional Unconstrained Solution

		Correlations of Factor Loadings			
		1	2	3	4
Mode A	1	1.00	0.86	0.08	0.39
	2	0.86	1.00	0.30	0.17
	3	0.08	0.30	1.00	0.46
	4	0.39	0.17	0.46	1.00
Mode B	1	1.00	-0.92	0.79	-0.65
	2	-0.92	1.00	-0.80	0.61
	3	0.79	-0.80	1.00	-0.72
	4	-0.65	0.61	-0.72	1.00
Mode C	1	1.00	0.86	0.30	0.47
	2	0.86	1.00	0.33	0.53
	3	0.30	0.33	1.00	0.34
	4	0.47	0.53	0.34	1.00

dimension three of the three-dimensional solution has become dimension four.) Note, a further splitting occurs in the five-dimensional solution to produce a subset of several dimensions for which all of the intercorrelations are above .8. In Mode *A*, there are only two members of this set, but in Mode *B* there are four and in Mode *C*, three. This pattern of degeneracy for the four- and five-dimensional solutions clearly indicates that one has gone beyond the number of dimensions that is capable of being distinguished by our sample of 34 subjects, at least by means of an unconstrained solution.*

Split-Half Evaluation of Reliability. The strongest evidence for the "reality" of a factor—namely, that it is due to systematic influences and not just random noise—is the demonstration that the same or similar versions of the factor can be found in several independent samples of data. To provide a check on the generalizability of our solutions over independent samples of data, split-half techniques were employed. The total sample was randomly divided into two subsamples of 17 each; for purposes of discus-

*At the time this manuscript was written, the theory of degenerate solutions had not been developed to the degree that is described in Harshman and Lundy (chapter 6). We would now consider the pattern in the four-dimensional solution to indicate a classic degeneracy with respect to dimensions one and two. Thus, we are not surprised that application of constraints allows additional dimensions to be recovered.

sion, these subsamples are labeled *R* and *S*. In order to guard against an "unlucky" split (for instance, one which by chance allocates most of the subjects who use a particular dimension into one of the two groups, and thus fails to find that dimension in both subsamples despite the "reality" of the dimension), a second division of the sample was made, roughly orthogonal to the first. For the second division, an even-odd respondent split was used and the resulting samples are labeled *E* and *O*. The total sample of $N = 34$ is labeled *T*.

Because our total sample consists of only 34 subjects, the split-half subsamples are somewhat smaller than one would conventionally utilize for reliability comparisons. With such small subsamples ($N = 17$), one may be able to verify the larger, more pronounced dimensions, but smaller, more subtle effects may be lost against the background noise. Nonetheless, we proceed with a description of the split-half comparisons both to provide what verification we can in this specific case and also to demonstrate in general how this important part of the analysis procedure is carried out. The reader should keep in mind, however, that failure to find replication of a factor in two split-halves, when each is based on only 17 subjects, may merely be an indication of too small a sample size to define the factor against the background noise. On the other hand, if a factor is found to replicate, this would provide strong evidence that the factor was the result of fairly sizable systematic effects generalizable across samples of subjects.

PARAFAC analyses in two through six dimensions were performed on the four subsamples, and the results were compared by computing correlations among dimensions. The two-dimensional solution was verified through the random split-half analyses. That is, similar solutions were obtained for each split-half of the random split, and each resembled the total group solution (Table C–7). Interestingly, however, the even-odd split did not verify the two-dimensional solutions. Comparison with the total sample solution (Table C–7) revealed that while both dimensions of the even split-half matched those of the total group solution, only one dimension of the odd split-half matched the total solution, and, surprisingly, this was different for Mode *A* versus *B*. Apparently, because of an "unlucky split", dimensions one and two were not as clearly the most important dimensions in the odd split, so the solution adjusted itself to bring in components of dimension three. However, for purposes of validation of a set of dimensions, "once is enough." If a given dimension were due to random noise, it should not replicate in any split-half comparison. Consequently, the *R* versus *S* cross-validation is adequate to verify the "reality" or nonrandom nature of the two dimensions of the two-dimensional solution. A more sophisticated comparison —such as to the three-dimensional solutions—is not needed.

We were surprised to find that the three-dimensional (unconstrained) solution was not validated by the split-half procedure, despite the fact that all three dimensions seemed highly interpretable (the interpretation will be discussed below). In *both* the *R* versus *S* and *E* versus *O* comparisons, one or more dimensions fail to correlate highly across the two halves of the data. Comparison of each split-half solution with the total group solution

TABLE C-7. Correlations for the Two-Dimensional Unconstrained Split-Half Analyses

Mode A

		S				O	
		1	2			1	2
R	1	.89	.29	E	1	.50	-.56
	2	-.42	.88		2	-.95	-.32

		T		R		S		E		O	
		1	2	1	2	1	2	1	2	1	2
T	1	1.00	.50	.95	-.52	.98	.45	.98	-.55	.53	-.57
	2	.50	1.00	.35	-.93	.40	.99	.47	-.96	.99	.42

Mode B

		S				O	
		1	2			1	2
R	1	.94	-.06	E	1	.67	-.92
	2	-.10	-.82		2	-.50	-.27

		T		R		S		E		O	
		1	2	1	2	1	2	1	2	1	2
T	1	1.00	-.26	.96	-.06	.99	-.17	.95	.20	.69	-.99
	2	-.26	1.00	-.18	-.85	-.22	.99	-.19	-.93	.51	.35

(labeled *T*) revealed that one of the two solutions in each split-half pair had broken down (Table C–8). While solutions *R* and *E* each contained all three dimensions of the total-group solution (although in different orders), their corresponding halves, *S* and *O*, showed various patterns of degeneracy. For Mode *A*, comparisons between the total group solution *T* and split-half solution *S* showed dimension *T*–1 correctly recovered as *S*–3, but *T*–2 split into two different dimensions, recovered as *S*–1 and *S*–2. This splitting of *T*–2 left no room for the third dimension (*T*–3) to emerge in the *S* split-half solution. Oddly, in Mode *B*, *T*–3 was represented; in fact, it had split into two dimensions and "crowded out" *T*–2. In some sense, we might claim that all three dimensions were replicated, but not in both modes simultaneously. It is perhaps more correct to say that the *S* split-half solution is in some sense degenerate. For the *E* versus *O* split, similar comments apply, with the *E* split replicating all three dimensions of the *T* solution, but the *O* split showing various patterns of nonreplication of particular dimensions in particular modes. (The *R* versus *E* comparison cannot be used to validate the reality of the dimensions, since they are not independent; half of their subjects are in common between the two samples.)

As might be expected, given the three-dimensional solutions,

TABLE C-8. Correlations for the Three-Dimensional Unconstrained Split-Half Analyses

Mode A

	S 1	S 2	S 3		O 1	O 2	O 3
R 1	-.25	-.11	.72	E 1	.92	-.51	.04
R 2	.95	.84	-.35	E 2	-.63	.09	-.80
R 3	-.36	-.00	-.05	E 3	-.22	-.48	-.31

	T 1	T 2	T 3	R 1	R 2	R 3	S 1	S 2	S 3	E 1	E 2	E 3	O 1	O 2	O 3
T 1	1.00	-.36	.28	.92	-.28	.20	-.45	-.37	.92	-.41	.38	.99	-.16	.10	-.06
T 2	-.36	1.00	-.44	-.17	.99	-.40	.98	.89	-.44	.98	-.72	-.40	.94	.56	-.40
T 3	.28	-.44	1.00	.42	-.48	.99	-.40	-.03	.02	-.28	.93	.28	-.47	.30	.87

Mode B

	S 1	S 2	S 3		O 1	O 2	O 3
R 1	-.25	.36	.95	E 1	.44	.13	-.42
R 2	.22	.09	-.23	E 2	.16	.30	-.87
R 3	-.62	.63	-.45	E 3	-.77	-.92	.48

	T 1	T 2	T 3	R 1	R 2	R 3	S 1	S 2	S 3	E 1	E 2	E 3	O 1	O 2	O 3
T 1	1.00	-.01	-.13	.98	-.32	-.44	-.17	.31	.99	.04	-.16	.99	-.83	-.97	.56
T 2	-.01	1.00	.27	-.09	.94	.33	.19	.19	.08	.92	.12	.08	.55	.18	-.25
T 3	-.13	.27	1.00	-.10	.31	.93	-.82	.86	-.16	.60	.98	-.05	.21	.29	-.88

the four- and five-dimensional split-half analyses showed even greater numbers of highly intercorrelated dimensions, with factors splitting into several copies of themselves and other degeneracies emerging. This was consistent with the behavior of the total-group solutions and indicated that four or more dimensions could not be recovered from this sample by means of unconstrained analyses.

The three-dimensional solution for the total group consisted of relatively uncorrelated, stable, and (as we shall see below) interpretable dimensions. The failure to recover these dimensions consistently when analyzing the split-half data sets suggested that perhaps 17 subjects was just too small a sample to recover three dimensions reliably. Since recent experience with other data indicated that premature emergence of degenerate solutions could be blocked by applying constraints to the form of the solution, it was decided to try constrained analyses of this "Cars and Stars" data set.

Constrained Analyses

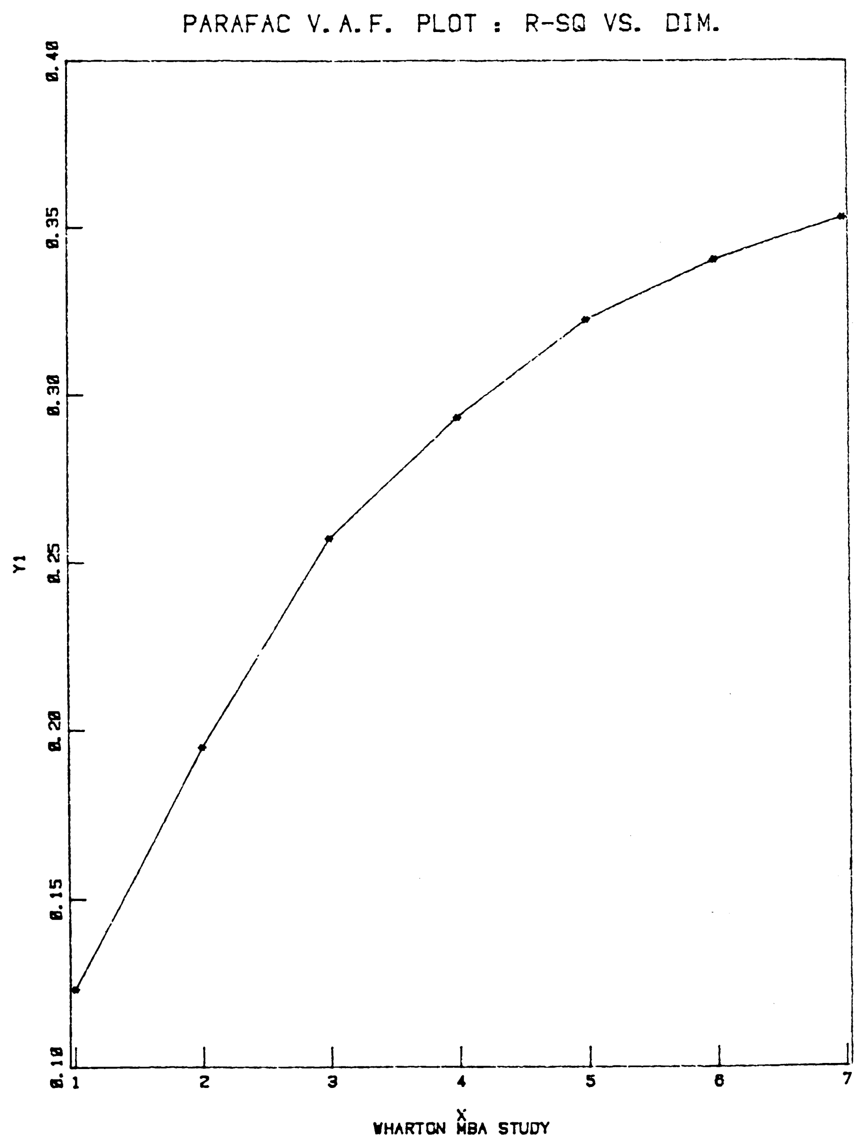
PARAFAC allows analyses to be performed subject to the constraints that the columns of factor loadings in a particular mode or modes be mutually orthogonal or mutually uncorrelated. By imposing such a constraint on our solutions, we hoped to block the emergence of the degeneracies in which a dimension splits into two highly correlated versions of itself. This would hopefully allow us to detect additional weaker dimensions that were previously obscured by a premature "breakdown" of the solution. The "reality" of these additional dimensions would then be tested by split-half methods and by consideration of their interpretability. It was hoped that with the constraint, 17 subjects might be sufficient to recover three reliable dimensions.

Constraining a single mode is often sufficient to block the emergence of highly correlated dimensions in all modes, provided that the correct mode is selected and that there is not some strong internal characteristic of the data promoting highly correlated factors. By constraining only one mode, the other two could take on whatever form was consistent with the data, and the solution would be a closer approximation to the "natural" unconstrained form. It was decided that Mode *B* (the rating-scale mode) should be constrained to be orthogonal, since (a) Mode *B* loadings were most orthogonal in the unconstrained three dimensional solution, and hence if a "real" replicable solution existed in both split-halves, it might not be distorted by requiring Mode *B* to be orthogonal; and (b) interpretation of the dimensions was primarily based on the scale loadings and such interpretation would be facilitated by keeping the patterns of scale loadings distinct for the different dimensions. It was not necessary to choose between orthogonality and zero-correlation constraints, since the centering of Mode *B* resulted in zero mean loadings in this mode and consequently both types of constraints become equivalent.

A series of constrained analyses was run in dimensionalities from one through seven. As before, three random starting positions were used in each analysis to check stability; checks of fit values, correlations among dimensions (Mode *A* and *C*), and comparisons of split-half solutions were used to evaluate the maximum valid dimensionality.

Fit Values. The R^2 values for the 1 through 7 dimensional constrained solutions were as follows: 1D = .123; 2D = .195; 3D = .257; 4D = .293; 5D = .322; 6D = .340; 7D = .353. Figure C–2 presents the plot of fit versus dimensionality based on R^2 or variance accounted for, as before. Once again, the evidence is somewhat ambiguous. In general, however, these data seem to suggest the presence of at least three dimensions, with perhaps a smaller fourth and possibly even fifth and sixth dimensions. The question may thus become not how many dimensions *there are,*

Figure C–2. Plot of Dimensionality versus R^2 for Constrained Solutions



but rather how many *can be reliably determined* on the basis of only 34 subjects, with 17 in each split-half data set.

Within-Solution Correlations among Dimensions. Naturally, the imposition of orthogonality constraints on Mode *B* prevents us from using this mode to check for factor-splitting and highly correlated dimensions. However, we can check Modes *A* and *C*, since these were not constrained. It turns out that the imposition of constraints on Mode *B* succeeded in suppressing factor-splitting degeneracies in all modes, for all the solutions that we considered, including four-, five-, and six-dimensional solutions. While this prevents us from using high correlations as a diagnostic of extracting too many factors, it does clear the way for the use of the more important split-half testing on the three-dimensional solution (and on higher dimensional solutions, if warranted).

Split-Half Evaluation of Reliability. For two- through six-dimensional solutions, split-half analyses were performed and compared. As before, the two-dimensional solution was verified by appearance in the various split-halves of the data; however, there was some difficulty in replicating dimension two for Mode *A* (Table C–9). For three dimensions, both the *R* versus *S* and the *E* versus *O* split-halves cross-validated the total-group solution for all three dimensions, thus confirming our suspicion that the third dimension was, in fact, "real" (Table C–10). Furthermore, the total-group three-dimensional constrained solution was almost identical to the three-dimensional unconstrained solution (Table C–11), which is not surprising since the unconstrained solution was close to orthogonal in Mode *B*. The near identity of the constrained and unconstrained solutions is nonetheless important to note, because it implies that the split-half confirmation of the constrained solutions in fact provides split-half validation of the three-dimensional unconstrained solutions, as well, and supports our notion that the failure to cross-validate by means of unconstrained analyses was probably due to too few subjects in each split-half sample.

To determine whether additional reliable patterns existed beyond the three dimensions obtained in the unconstrained solutions, we examined correlations between split-half solutions in four through six dimensions. Tables C–12, C–13, and C–14 present the relevant cross-split correlations.

Somewhat surprisingly, the comparison of split-halves provides evidence for additional reliable dimensions beyond the third. As Table C–12 indicates, a constrained four-dimensional solution is cross-validated by clear replication in split-halves. While the order of dimensions three and four is reversed in sample *S* relative to sample *R* (indicating a reversal of their relative importance in terms of variance accounted for in the two subsamples), there is only one large correlation in each row and column of the matrix of cross-solution correlations. Furthermore, all these correlations are high. For Mode *A* they are .89, .92, -.88, and .92; for Mode *B* they are .96, .93, -.94 and .89. (The occurrence of a negative correlation for dimension three simply means that it was reflected in solution *S* relative to *R*.) The *E* versus *O* split also provides support for four dimensions, although some of the correlations are not as high as in the *R*-*S* split, and in fact drop

TABLE C-9. Correlations for the Two-Dimensional Constrained Split-Half Analyses

Mode A

		S				O	
		1	2			1	2
R	1	.83	.60	E	1	.92	-.34
	2	-.27	.74		2	.48	.60

		T		R		S		E		O	
		1	2	1	2	1	2	1	2	1	2
T	1	1.00	.30	.89	-.27	.98	.26	.89	.03	.81	-.66
	2	.30	1.00	.66	.71	.30	.99	.61	.79	.79	.49

Mode B

		S				O	
		1	2			1	2
R	1	.85	.46	E	1	.91	-.26
	2	-.40	.80		2	.30	.79

		T		R		S		E		O	
		1	2	1	2	1	2	1	2	1	2
T	1	1.00	.00	.85	-.42	.99	-.03	.83	-.26	.73	-.67
	2	.00	1.00	.51	.73	.03	.99	.47	.76	.67	.72

as low as .81 and .85. In Mode *A*, the *E* versus *O* matrix of cross-split correlations shows additional large correlations in rows 2 and 3, indicating, perhaps, a less stable axis position for the second and third dimensions in the *E* versus *O* split. Overall, however, these results provide strong support for the presence of four reliable dimensions.

Although comparison of the five-dimensional analyses of the random (*R* versus *S*) split-halves of the data (Table C–13) provides some support for a fifth dimension, the cross-sample correlation for dimension five is fairly low. In the *R* versus *S* comparison, it is .78 for Mode *A* and .74 for Mode *B*. In the *E* versus *O* comparison, the lowest cross-sample correlation (taking into account the reordering of dimensions) is .66 in Mode *A* and .67 in *B*. It may be that we have reached the limit of the number of dimensions that can be adequately cross-validated with this data and this size sample. Also, in the *E* versus *O* comparison, the plane involving dimensions two and four of each solution seems to have some rotational ambiguity, since the four cross-split correlations resemble an attenuated rotation matrix (first row = -.50, .67, second row = .69, .63). No such rotational ambiguity is indicated in the *R* versus *S* comparison correlations, however.

TABLE C-10. Correlations for the Three-Dimensional Constrained Split-Half Analyses

Mode A

	S 1	S 2	S 3
R 1	.90	.25	-.33
R 2	-.28	-.29	.96
R 3	-.06	.93	-.03

	O 1	O 2	O 3
E 1	.96	.20	-.02
E 2	-.36	-.89	.15
E 3	.27	.47	.90

	T 1	T 2	T 3	R 1	R 2	R 3	S 1	S 2	S 3	E 1	E 2	E 3	O 1	O 2	O 3
T 1	1.00	-.32	.14	.97	-.30	-.02	.98	.16	-.31	.99	-.22	.16	.98	.14	-.04
T 2	-.32	1.00	-.20	-.35	.99	-.08	-.31	-.28	.99	-.37	.98	-.31	-.47	-.94	-.02
T 3	.14	-.20	1.00	.25	-.20	.98	.11	.98	-.14	.16	-.01	.98	.23	.38	.96

Mode B

	S 1	S 2	S 3
R 1	.96	.13	-.03
R 2	.03	-.04	.97
R 3	-.13	.95	.04

	O 1	O 2	O 3
E 1	.98	-.02	.01
E 2	-.02	-.88	.29
E 3	-.02	.29	.92

	T 1	T 2	T 3	R 1	R 2	R 3	S 1	S 2	S 3	E 1	E 2	E 3	O 1	O 2	O 3
T 1	1.00	.00	.00	.98	.03	-.12	.99	.01	-.00	.98	.12	-.09	.98	-.15	-.04
T 2	.00	1.00	.00	-.03	.99	.01	.00	-.04	.99	-.12	.97	-.14	-.14	-.94	.16
T 3	.00	.00	1.00	.12	.00	.98	-.01	.99	.04	.06	.15	.98	.05	.17	.97

TABLE C-11. Correlations between the Three-Dimensional Constrained and Unconstrained Total Group Solutions

Mode A

		Orthogonal		
		1	2	3
	1	1.00	-.35	.23
Unconstrained	2	-.32	.99	-.33
	3	.17	-.29	1.00

Mode B

		Orthogonal		
		1	2	3
	1	1.00	-.01	-.01
Unconstrained	2	.02	1.00	.09
	3	.11	.16	.98

TABLE C-12. Correlations for the Four-Dimensional Constrained Split-Half Analyses

Mode A

		S						O			
		1	2	3	4			1	2	3	4
	1	.89	-.40	.06	-.36		1	.91	-.13	-.08	-.00
R*	2	-.29	.92	-.36	.25	E	2	-.61	-.47	.10	.81
	3	.15	.23	.28	-.88		3	-.23	-.74	-.88	-.02
	4	.12	-.36	.92	-.20		4	.09	.92	.25	.11

Mode B

		S						O			
		1	2	3	4			1	2	3	4
	1	.96	-.03	-.03	-.20		1	.93	-.21	.01	.19
R*	2	.01	.93	-.08	.12	E	2	-.26	-.41	.08	.81
	3	-.19	.13	.13	-.94		3	-.02	-.10	-.94	.01
	4	-.00	-.02	.89	.10		4	.08	.85	-.03	.44

*~~Indicates~~ best solution of three (largest R^2)

Correlations among six-dimensional analyses of the split-half subsamples (Table C–14) indicate that we cannot recover six reli-

TABLE C-13. Correlations for the Five-Dimensional Constrained Split-Half Analyses

Mode A

		S 1	S 2	S 3	S 4	S 5			O 1	O 2	O 3	O 4	O 5
	1	.90	.11	-.36	-.37	-.13		1	.86	-.42	.07	-.01	.08
	2	.26	.92	-.47	-.17	.18		2	-.62	-.39	-.28	.66	.18
R*	3	.19	.06	.83	-.35	-.18	E	3	.45	.19	-.80	-.07	-.44
	4	-.28	-.45	.21	.89	-.02		4	-.07	-.77	.25	-.51	.31
	5	.02	.31	-.17	-.36	.78		5	-.15	-.26	.01	.10	.92

Mode B

		S 1	S 2	S 3	S 4	S 5			O 1	O 2	O 3	O 4	O 5
	1	.96	-.11	-.19	-.06	-.01		1	.86	-.44	.01	.15	.01
	2	.07	.90	-.20	.23	.02		2	-.38	-.50	.22	.67	-.01
R*	3	.19	.17	.93	-.11	-.06	E	3	.09	.12	-.90	-.10	-.25
	4	.01	-.22	.09	.84	.15		4	.22	.69	.02	.63	.02
	5	-.05	.10	-.03	-.33	.74		5	-.01	.00	.21	-.03	.92

*Indicates best solution of three (largest R^2)

able dimensions. Evidently, such analyses have gone beyond the limit of what we can resolve with this data, even using orthogonality constraints.

Given the somewhat gradual "trailing off" of our ability to recover additional dimensions, the question arises as to which dimensionalities we should consider in detail. If this "Cars and Stars" study were a serious attempt to evaluate spokesmen for car makes, rather than simply an example of how this technique might work, we would probably consider the four- and five-dimensional solutions quite carefully. We would also interpret the indications of higher dimensionality as evidence for the need to repeat the study on a larger sample, so as to better define the additional dimensions. (However, there are also other aspects of the study that would need to be improved at the same time; this 34-subject "Cars and Stars" study might best be viewed as a preliminary or pilot study.)

For purposes of demonstrating how we arrive at interpretations of dimensions, we have decided to concentrate on the simpler three-dimensional unconstrained solution, rather than emphasize the refinements suggested by the smaller higher dimensions. Some brief discussion of the four-dimensional solution will be presented, however, to provide some insight into the type of information that higher dimensions might add to our basic solution.

TABLE C-14. Correlations for the Six-Dimensional Constrained Split-Half Analyses

Mode A

		S								O					
		1	2	3	4	5	6			1	2	3	4	5	6
	1	.88	-.26	-.46	-.25	-.02	-.37		1	.93	.30	.09	.03	-.27	-.08
	2	-.39	-.93	-.37	-.31	.21	.26		2	-.53	-.64	-.18	.82	-.24	.40
R*	3	-.12	.20	-.68	.27	-.20	.83	E	3	-.23	-.45	.80	-.16	-.26	.32
	4	.03	-.31	-.45	-.33	-.58	-.33		4	-.12	.65	-.30	-.09	.31	.59
	5	-.29	.02	.68	-.43	.08	-.10		5	.12	-.05	.23	-.11	-.87	.01
	6	.22	-.09	.27	-.67	.79	-.01		6	-.17	-.73	-.25	.04	.51	.06

Mode B

		S								O					
		1	2	3	4	5	6			1	2	3	4	5	6
	1	.93	-.14	-.19	-.18	-.08	.05		1	.95	.08	.00	.21	-.12	.01
	2	.08	.86	.18	-.34	-.13	-.19		2	-.16	-.41	-.11	.73	-.18	.36
R*	3	-.12	.05	-.50	-.43	.68	-.06	E	3	.05	-.32	.81	-.23	-.09	.37
	4	-.17	-.40	.20	-.59	-.31	-.44		4	.06	.53	-.20	-.09	-.01	.76
	5	-.13	-.02	.07	-.45	-.20	.82		5	-.08	.14	.02	-.15	-.89	-.10
	6	-.19	.08	-.77	.03	-.49	-.03		6	.12	-.58	-.47	-.41	-.16	.18

*~~Indicates~~ best solution of three (largest R^2)

Interpretation of Dimensions

The Role of Interpretation

Interpretation of the PARAFAC dimensions plays a role in several stages of the analysis. Preliminary scanning of the dimensions for interpretability provides some useful guidance in the earlier phases of analysis. With the "Cars and Stars" data, for example, the apparent interpretability of all three dimensions of the unconstrained three-dimensional solution was one of the reasons we were reluctant to reject the solution when split-half comparisons did not validate it. (Another reason was the suggestion of a third dimension in most fit versus dimensionality curves.) However, these preliminary attempts at interpretation are not nearly as crucial as the stage of careful and detailed interpretation that is required once a preferred dimensionality (or set of dimensionalities) is selected.

The goal of *connotative congruence analysis,* as applied to our marketing example, is to select an effective spokesman-product relationship. As we shall see, this will often require artful design of the spokesman's message so that it picks out particular desired aspects of the spokesman's overtones and relates them in the most fruitful way to those properties of the product that one wants to enhance. To construct such messages requires the best possible understanding of the dimensions of perceived overtones for a particular spokesman and product, as indicated by the PARAFAC analysis.

The Method of Interpretation

As mentioned earlier, one advantage of using PARAFAC for this type of study is that it provides direct estimates of loadings for all three modes—the spokesman-product mode, the rating scale mode, and the rater mode. However, it is the rating scale mode (Mode *B* in our data set) that provides the primary basis for interpretation of the obtained factors or dimensions. For each dimension, there should be some rating scales with particularly high (positive or negative) loadings. Furthermore, the scales with high loadings on a given dimension should have one or two overtone elements in common. It is these common elements that primarily define the "meaning" represented by that dimension.

When interpreting the sizes and signs of the ratings scale loadings, two points should be kept in mind. First, those scales with the highest weights on any given dimension should be the ones that most strongly exemplify the underlying meaning of the dimension; as the loadings get smaller, the scales should show a progressively weaker relationship to the common element of meaning or "overtone" represented by the dimension. Second, both ends of any bipolar rating scale should be considered when assessing its relationship to the "meaning" of a dimension. For a scale that loads positively on a given dimension, the word from the high end of the scale should be used to interpret the positive pole of the dimension, and the opposite-meaning word, from the low end of the rating scale, should be used to interpret the negative pole of the dimension. For scales with negative loadings, the situation is reversed—words from the high end of the scale contribute to interpretation of the negative pole of the dimension, and words from the low end of the rating scale contribute to the positive pole.

After considering the rating scale loadings for a given dimension, one can obtain additional confirmation and/or further refinement of one's interpretation by examining the loadings in Mode *A* to determine the common properties of those spokesmen and products that have high loadings on that dimension. In addition, if one has demographic, psychographic, or other relevant information on raters or market segments, such information can be related to the Mode *C* loadings to provide another check on the sensibleness of one's interpretation.

Three-Dimensional Unconstrained Solution

The loadings for the three-dimensional unconstrained solution are presented in Tables C–15, C–16, and C–17. To aid in our discussion of this solution, we will use a graphical representation of each dimension, which provides a visual summary of the relative strength of different items that load substantially on the dimension. Figure C–3 diagrams the loading patterns for the first dimension of the three-dimensional solution. Information on all

TABLE C-15. Mode A for Three-Dimensional Unconstrained Solution

	Mode A: Cars & Stars & Self		
	1	2	3
1	-0.78	0.12	0.16
2	-0.37	0.27	-1.71
3	-1.77	-0.72	-1.51
4	-0.24	-1.16	2.84
5	-0.21	0.35	-2.19
6	-0.60	1.58	-1.20
7	-0.77	-0.96	0.09
8	-0.15	0.28	-0.01
9	-0.14	-0.84	-0.27
10	-0.15	-1.71	0.49
11	-0.04	0.37	-0.61
12	-0.72	-0.33	1.77
13	1.18	-0.26	0.25
14	0.18	0.90	0.22
15	0.53	-0.47	0.45
16	-1.18	2.15	-0.35
17	0.21	0.68	0.13
18	2.31	-1.65	0.62
19	0.93	0.96	-0.11
20	1.70	-0.91	0.24
21	2.16	-0.21	0.39
22	-1.37	2.08	-0.47
23	0.03	-0.48	0.28
24	0.03	-0.21	0.37
25	-0.74	0.17	0.09

Note: Factor one loadings should be reflected (reversed in sign) to be consistent with the discussion and Figures 24-26.

three modes is presented on the diagram. To save space, only items with moderate to high loadings are plotted, leaving bare the crowded area around the zero-point where items unrelated to the dimension are found. To further save space, the contribution of a given rating scale (in the plot of Mode *B* loadings) is usually represented by only its high-end adjective, rather than following the more correct procedure of plotting its high-end adjective on one pole of the dimension and its low-end adjective on the other. (In the text, however, the contribution of the unplotted complimentary adjectives will be mentioned, along with the plotted ones.) Note that in Figure C–3, the signs of loadings for Mode *A* and *B* have been reflected to simplify discussion. In effect, we simply reverse the poles of the dimension and call it "flashiness" instead of "plainness." (All loadings in Figures C–3 through C–5 have been multiplied by 100 for convenience.)

TABLE C-16. Mode B for Three-Dimensional Unconstrained Solution

	Mode B: Rating Scales		
	1	2	3
1	0.70	-0.70	-1.79
2	0.67	-0.55	1.18
3	-0.04	-1.91	-0.38
4	1.24	1.22	0.53
5	-1.43	-0.38	-0.84
6	-1.81	-0.22	-0.35
7	1.14	0.11	1.05
8	-1.57	0.66	0.57
9	0.76	-1.33	-0.31
10	1.07	-0.27	-1.27
11	-0.89	1.11	-0.19
12	-0.27	1.20	-0.63
13	-0.62	-1.79	0.49
14	-1.06	-1.32	0.58
15	0.79	0.04	-0.30
16	1.12	-0.18	-0.81
17	-0.42	1.39	-0.21
18	1.03	0.84	1.29
19	-0.13	0.83	1.91
20	1.36	1.62	1.03
21	0.70	-0.77	0.97
22	-0.03	0.38	2.14
23	-1.49	-0.86	-0.35
24	1.28	-0.60	0.07
25	-1.42	0.65	1.19
26	-0.87	1.46	-0.30
27	0.24	-0.44	-0.81
28	-1.31	1.04	0.99
29	0.58	-1.10	-2.03
30	0.16	1.30	0.39
31	-1.56	0.10	-0.21
32	-0.90	-1.12	0.55
33	-0.98	-1.08	-0.92
34	1.33	-0.23	-1.52
35	0.25	0.84	0.50
36	0.03	1.66	-1.10
37	1.06	0.19	0.01
38	0.01	-1.36	0.65
39	1.25	-0.40	-1.74

Note: Factor one loadings should be reflected to be consistent with the discussion and Figures 24-26.

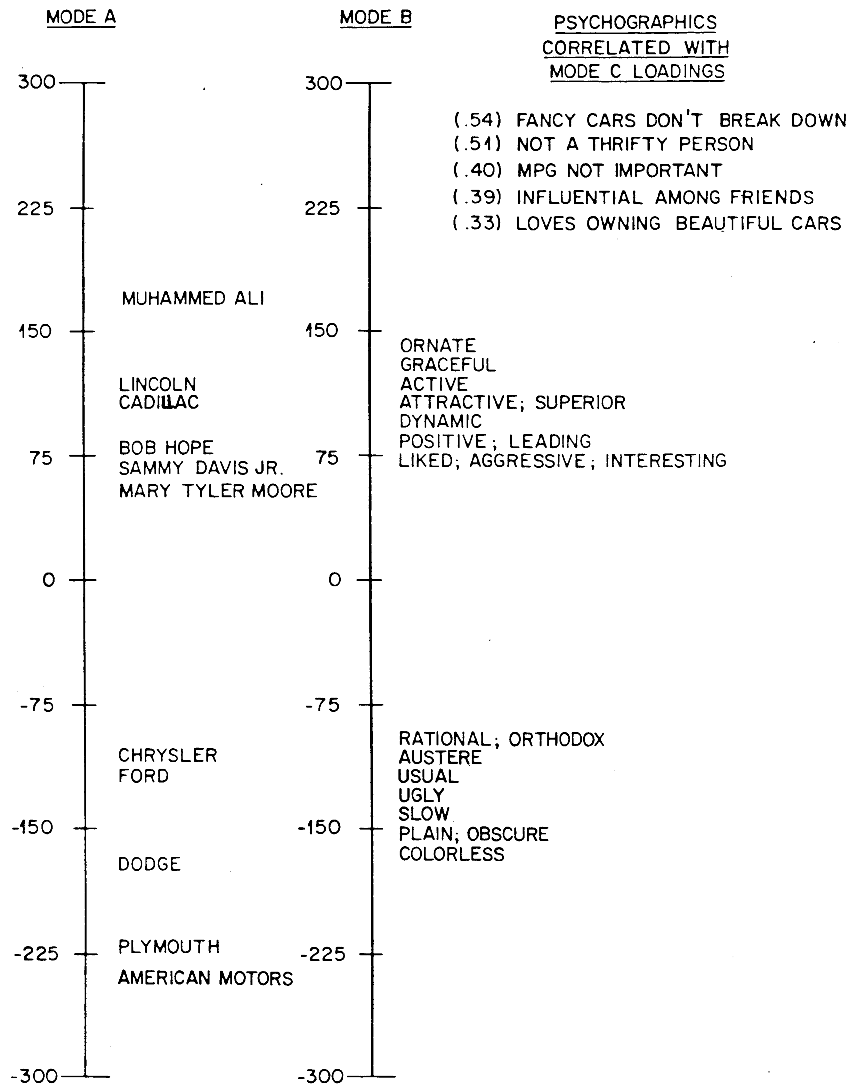
To interpret dimension one, we focus first on the Mode *B* loadings. Words such as "colorful," "famous," "ornate," "grace-

TABLE C-17. Mode C for Three-Dimensional Unconstrained Solution

	Mode C: Subjects		
	1	2	3
1	0.36	0.26	0.30
2	0.34	0.34	0.36
3	0.34	0.46	0.25
4	0.18	0.25	0.36
5	0.41	0.31	0.26
6	0.11	0.53	0.36
7	0.29	0.22	0.29
8	0.38	0.27	0.41
9	0.22	0.26	0.29
10	0.16	0.39	0.30
11	0.38	0.34	0.25
12	0.33	0.37	0.24
13	0.42	0.24	0.18
14	0.29	0.36	0.32
15	0.46	0.29	0.29
16	0.24	0.41	0.28
17	0.12	0.14	0.21
18	0.06	0.33	0.27
19	0.34	0.35	0.15
20	0.54	0.23	0.22
21	0.37	0.32	0.26
22	0.51	0.21	0.21
23	0.35	0.31	0.33
24	0.13	0.22	0.20
25	0.05	0.15	0.25
26	0.35	0.21	0.14
27	0.43	0.13	0.22
28	0.29	0.29	0.38
29	0.45	0.18	0.27
30	0.20	0.43	0.37
31	0.20	0.09	0.21
32	0.28	0.20	0.30
33	0.25	0.21	0.17
34	0.35	0.23	0.35
Root-Mean-Squared Loading (Mode C) for Each Factor			
	0.3281	0.3010	0.2848

ful," "active," "attractive," and "superior" are found at the positive end of this dimension, with their complements "colorless," "obscure," "plain," "awkward," "passive," "unattractive," and "inferior" at the negative pole. Something perceived as high on this dimension is not just attractive and superior; it is even more

Figure C–3. Dimension One: "Flashy"



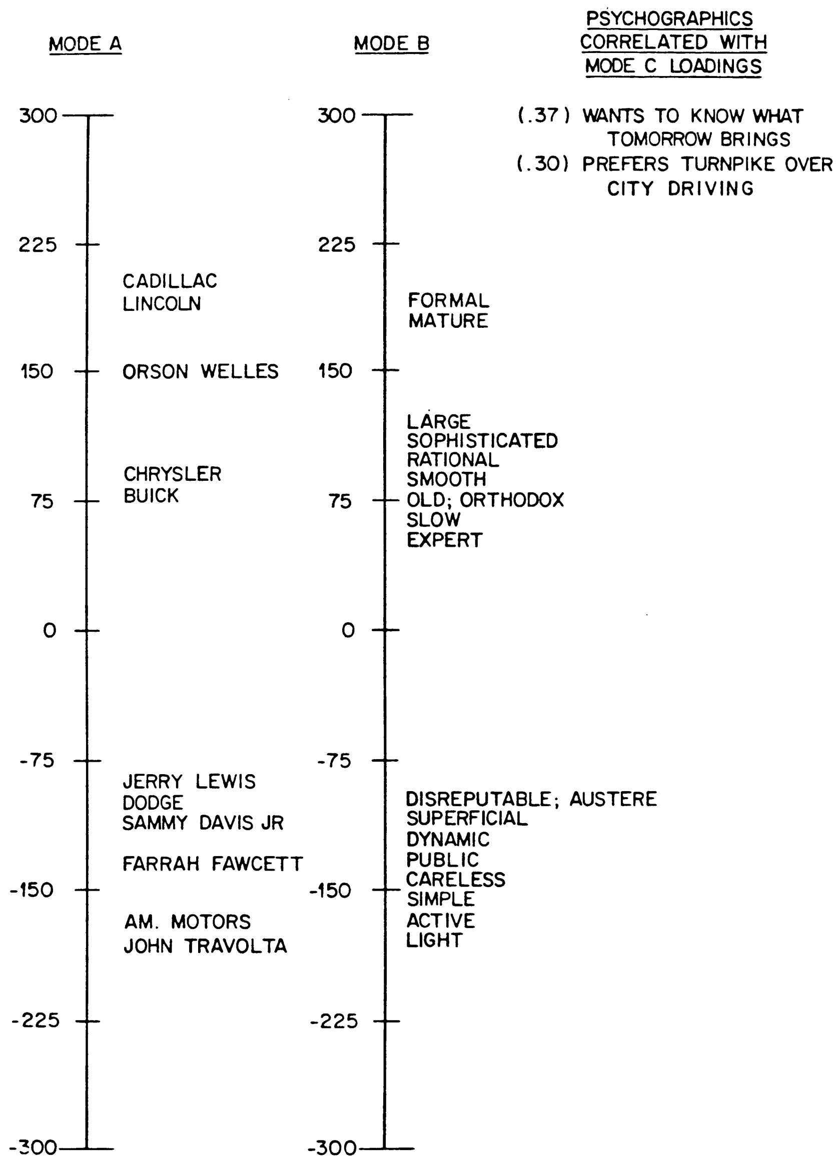
importantly famous and colorful, even ornate. Apparently, we are not just pleased by such a stimulus but impressed and fascinated as well. This interpretation of dimension one is consistent with the patterns of relationships found in its Mode *A* loadings, where Muhammed Ali, a very "charismatic" individual, has the highest loading (the survey was taken during the period when he was world champion and very popular). Both Ali and Sammy Davis, Jr., are also somewhat "flashy" or "ornate" in their public relations style. And while Bob Hope may not be "flashy" in the same way, he is colorful and very famous. Indeed, fame or renown must play a large role in this dimension, since all the celebrities load positively on it. In terms of products, the positive pole of this dimension is characterized by two automobiles that load much higher than any others: Lincoln and Cadillac. It is quite plausible that our sample of raters (MBA students) regarded these two makes as imbued with a special aura of impressiveness, a special symbolic renown. And the plush appointments of these automobiles might be considered "ornate" by some; certainly they are far from "plain." From a slightly different perspective, the possession of one of these makes might be considered a "flashy" and impressive gesture—appropriate for Sammy Davis, Jr., or Muhammed Ali. On the opposite end of the Mode *A* scale, we find American Motors and Plymouth; these names certainly do not have the same kind of "charisma" as Lincoln and Cadillac.

In light of all the interpretive relationships noted above, we will use the word "flashy" as a shorthand label for the overtone pattern expressed by this first dimension. The reader should avoid, however, the association of cheapness or tinsel-falseness that some might associate with the word "flashy." We mean to evoke something like impressive, colorful, renowned, dramatic, "showy."

As a further source of insight into this dimension, we can examine the characteristics of individuals for whom this dimension is important. Mode *C* loadings (after row normalization) give the relative importance of the three dimensions for each subject. Correlating the dimension one (Mode *C*) loadings with responses to each of the psychographic questions reveals whether individuals for whom "flashiness" or impressive and colorful renown is highly salient tend to answer certain questions differently than those for whom the flashiness dimension is unimportant. We find that people who tended to ignore this dimension of "flashiness" when rating cars and celebrities tended to be unimpressed by the mechanical properties of fancy cars. They were more likely to agree that "fancier, more expensive cars, probably break down a lot" ($r = .54$). They would also tend to be thrifty ("Those who know me would consider me to be a thrifty person" [$r = .51$] and would agree that "miles per gallon statistics are very important to me in my selection of a new car" ($r = .40$). It is quite plausible that such persons would not pay much attention to the "flashy" dimension of automobiles (or people). On the other hand, since "flashy" cars are expensive to purchase and to run, it is reasonable that those who find "flashiness" very salient do not care as much about financial considerations and consider big cars reliable. Such patterns of attitude would serve to reduce the cognitive dissonance that would arise from simultaneously longing after a "flashy" car and at the same time realizing that it was too expensive and unreliable. At a more general level, we might speculate that individuals who are unimpressed by "flashiness" in general (such as those who would be unimpressed by this quality of Muhammed Ali or Sammy Davis, Jr.) would tend to be more practically oriented and perhaps thrifty. Of course, these latter comments are highly speculative and so should be tested in a subsequent study before being taken too seriously.

Figure C–4 presents a graphical summary of the loading patterns for dimension two. Considering first the Mode *B* loadings, we find that the positive pole of this dimension was high loadings for "formal," "mature," "heavy," "passive," "complex," "large," "careful," and "sophisticated," while the negative pole loads on

Figure C–4. Dimension Two: "Mature/Conservative"



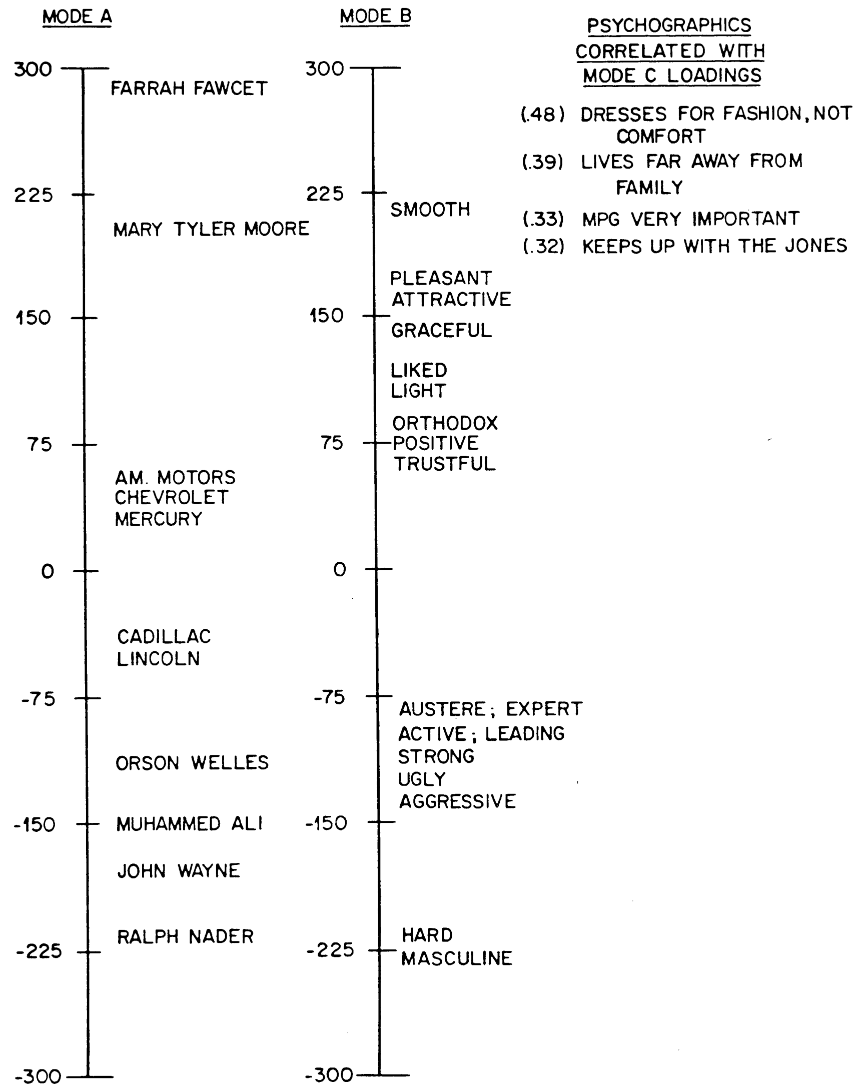
"informal," "youthful," "light," "active," "simple," "small," "careless," and "naive." This seems to be a dimension of maturity, heaviness, and formality. Looking at Mode *A* to confirm our interpretation, we find that Orson Welles has by far the largest positive loading of any celebrity. Compared to other celebrities, he would indeed be considered more mature, formal, and heavy. John Wayne and Ralph Nader have loadings near zero (not shown on Figure C–4; refer to Table C–15). On the opposite pole we have John Travolta, Farrah Fawcett, Sammy Davis, and Jerry Lewis, all of whom seem particularly youthful, informal, active, and arguably "light." These Mode *A* loading patterns confirm our interpretation of dimension two. In terms of automobile makes, there is another sharp contrast: Cadillac and Lincoln are similar to Orson Welles in terms of formality, maturity, and heaviness, with Chrysler and Buick also tending to be viewed as somewhat formal. On the other pole, American Motors is the most informal, youthful, and light of the makes, with Dodge showing the same overtones but to a considerably smaller degree.

Finally, if we examine the correlations of the psychographic responses of our raters with their weights on dimension two, we find that the individual for whom this dimension is highly salient "wants to know what tomorrow has in store for him" ($r = .37$) and "prefers turnpike to city driving" ($r = .30$). The person to whom this dimension is not important presumably lives more for today. These correlations suggest that the individual to whom dimension two is particularly salient may tend to be more conservative and prefer the mature, formal, and careful to the immature, informal, and careless. Consequently, we have named this dimension "Mature/Conservative" as a shorthand for the qualities that we surmise might be important to one who values the positive end of this dimension.

Dimension three (Figure C–5) most strongly emphasizes "feminine," "soft," and "smooth" on the positive pole versus "masculine," "hard," and "rough" on the negative pole. At a slightly lower level, words such as "pleasant," "attractive," and "graceful" load on the positive pole, with their opposites loading on the negative pole. This dimension seems to capture the sensual and aesthetic qualities associated with the feminine-masculine distinction. Clear confirmation of this is provided by the Mode *A* loadings, where Farrah Fawcett and Mary Tyler Moore are at the positive pole and Ralph Nader, John Wayne, Muhammed Ali, and Orson Welles at the negative pole. The most sensuous female has the highest positive loading, with Mary Tyler Moore quite a bit lower and Barbara Walters rated least "feminine" of all. It is surprising, perhaps, that Ralph Nader is viewed as most "masculine," but his emphasis of the critical, rational, sophisticated aspect here associated with masculinity may be responsible for this. The ranking of the other male celebrities is as we might expect, with John Wayne and Muhammed Ali being high, and some male entertainers such as Jerry Lewis, Bob Hope, and Sammy Davis being rated much lower. The automobiles all have fairly weak positive or negative loadings on this dimension. This seems plausible enough, since any sensual and gender related overtones of automobiles will surely be much less dramatic than those of celebrities such as are rated in this study. Although subtler, these automobile overtones appear to be modestly reliable (as indicated by comparison of loadings across split-half solutions) and consequently might be useful.

Individuals for whom this dimension is particularly salient tend to endorse the following psychographic questions: "When I must choose between the two, I usually dress for fashion, not comfort" ($r = .48$), and "I admit that I try to keep up with the Joneses" ($r = .32$). This suggests that the person is somewhat status-conscious or perhaps is concerned about appearances. The person also tends to "live a long way from friends and relatives"

Figure C–5. Dimension Three: "Feminine, Soft, Smooth"



(r = .39) and feels that "miles-per-gallon statistics are very important to me in my selection of a car" (r = .33). It is not clear as to why these last two psychographics should correlate with high salience of this dimension.

The third dimension contains several different threads of "Femininity-Masculinity," which might not always coincide in particular cases. Orson Welles might be rated relatively masculine for a different reason than Muhammed Ali. Similarly, Lincoln and Cadillac might be rated more masculine than Chevrolet and American Motors because of size or weight rather than appearance. One can always check back to the individual ratings of the stimuli (averaged across raters) to test for particular scale by stimulus interactions that might deviate from the general pattern indicated by the dimension as a whole. Furthermore, we have evidence from the appearance of small fourth and fifth dimensions, which pull some of these threads apart, that there are aspects of dimension two and three that are not as cohesive with the dimension as a whole, as the three-dimensional solution might suggest.

In addition to the factor loadings themselves, there are other outputs from the PARAFAC analysis that can aid in interpretation. We cannot take time to discuss all of these here, but will briefly mention one example. Table C–18 presents the error analysis table from the PARAFAC three-dimensional unconstrained analysis of the total group. The figures in the table are the mean-squared error values for each level of each mode of the data. By examining these values, one can determine whether certain parts of the data were better described by the PARAFAC model than others. This is particularly straightforward for Modes *B* and *C*, where the data were standardized so that each level has a mean square value of 1.0. Thus, for these two modes, the numbers in the error analysis table represent the variance not accounted for at each level—that is, for each stimulus (Mode *B*) and for each rater (Mode *C*). Examination of these tables reveals, for example, that rating scales 6, 8, 20, 26, and 36 were particularly well fit by the model. These represent the adjectives "colorful," "ornate," "active," "simple," and "heavy" along with their opposites. On the other hand, judgments on rating scales, 15, 27, 30, 35, and 37 were particularly poorly fit by the model. These scales contain the adjectives "familiar," "trustful," "private," "inefficient," "boring." Some scales that are very poorly fit in three dimensions might become better fit by the smaller additional dimensions that could emerge from a four- to six-dimensional analysis of a larger data set. Other scales might need companion scales to define a dimension more clearly. "Inefficient versus efficient" is an example of a scale that relates clearly to automobiles but not so obviously to celebrities; it also is somewhat isolated and has no closely related adjectives to help it define a dimension. The fact that only 6% of the variance of this rating scale was explained by the three-dimensional analysis (leaving a mean-square error of .94) is consistent with the idea that we were here concentrating on major connotative dimensions. If we would want, in a subsequent analysis, to define some less connotative and more directly descriptive scales, we might want to elaborate "Inefficient-Efficient" into a small cluster of scales related to fuel efficiency and extract additional small dimensions from our data until this dimension emerges (if it does). However, such a dimension would presumably have only modest loadings, at best, on the celebrities (although Ralph Nader, for example, might get relatively high loadings for efficiency and Jerry Lewis for inefficiency).

The Mode *C* mean-square error values from the error analysis table could be useful in determining whether the judgments of certain types of individuals—such as certain segments of the market—were better fit by the model than those of other individuals and/or market segments. One would know, then, which types of generalizations to trust more and perhaps would want to modify the rating scales or stimuli to better tap those segments that were not well accounted for in this analysis. With our data, for example, we see that the ratings of subjects 24, 25, 31, and

TABLE C-18. Error Analysis for Three-Dimensional Unconstrained Solution

Mode A		Mode B		Mode C	
1	0.794	1	0.749	1	0.722
2	0.925	2	0.707	2	0.667
3	0.970	3	0.708	3	0.628
4	0.786	4	0.772	4	0.745
5	0.925	5	0.707	5	0.683
6	0.694	6	0.633	6	0.607
7	0.756	7	0.732	7	0.794
8	0.635	8	0.665	8	0.637
9	0.783	9	0.742	9	0.808
10	0.901	10	0.803	10	0.753
11	0.988	11	0.715	11	0.696
12	0.638	12	0.745	12	0.714
13	0.737	13	0.671	13	0.742
14	0.439	14	0.753	14	0.700
15	0.586	15	0.936	15	0.632
16	0.926	16	0.846	16	0.714
17	0.578	17	0.737	17	0.926
18	0.708	18	0.754	18	0.825
19	0.717	19	0.748	19	0.746
20	0.671	20	0.662	20	0.616
21	0.544	21	0.702	21	0.709
22	0.738	22	0.674	22	0.661
23	0.605	23	0.759	23	0.690
24	0.638	24	0.735	24	0.899
25	0.866	25	0.693	25	0.918
		26	0.595	26	0.818
		27	0.949	27	0.751
		28	0.673	28	0.708
		29	0.673	29	0.697
		30	0.874	30	0.666
		31	0.705	31	0.910
		32	0.815	32	0.803
		33	0.812	33	0.871
		34	0.708	34	0.715
		35	0.944		
		36	0.532		
		37	0.887		
		38	0.739		
		39	0.680		

33 were particularly poorly fit by this analysis, whereas the ratings of subjects 3, 6, 15, and 20 were particularly well described by the PARAFAC three-dimensional analysis. If we knew enough about these particular subjects, we might be able to draw useful conclusions about what type of individual our analysis best describes.

The levels of Mode *A* were not rescaled to a constant mean-square as part of the preprocessing for this analysis, because it was thought useful to allow those stimuli that elicited stronger feelings to maintain this greater importance in the analysis and thus have larger resulting loadings. Consequently, the relative sizes of the mean-square error values for Mode *A* would have to be divided by their mean-square values at input to obtain a more appropriate idea of relative variance accounted for. Such information would be useful, for example, if it indicated that judgments relating to certain products of importance to the study were not at all well fit by the model. Generalizations with respect to those products should then be made with much greater caution. Alternatively, one might want to deliberately extract more factors (as long as reliable factors could be obtained) examining the error analysis table at each dimensionality to see if the additional dimensions provide for a better account of the products in question.

Four- and Five-Dimensional Solutions

The loadings for Mode *A* and *B* of the four-dimensional constrained solution are presented in Tables C–19 and C–20. The interpretation of dimension one stays more or less the same as in the three-dimensional solution. (Once again, we reflect the loadings on Modes *A* and *B* of dimension one for purposes of interpretation.) Dimension two is similar, but not identical. While it retains its formal, large, heavy, lush qualities, the "mature versus youthful" overtone component is no longer as strong. This component is now part of a new third dimension, along with the "rational-irrational," "careful-careless," and "profound-superficial" overtones previously associated with the masculinity-femininity dimension (dimension three). Since the new dimension three has at its positive pole the terms "youthful," "naive," "superficial," "irrational," "careless," and "disreputable," we might call it "impetuous youth" versus "responsible-thoughtful-maturity." Dimension four is a simplified version of the "feminine-soft-smooth" dimension. It no longer has the component of irrationality, carelessness, or superficiality associated with femininity. On the other hand, "pleasant" now has an even higher loading and consequently might be considered to be a clearer expression of the sexy-sensual-feminine versus macho-masculine dimension. Indeed, because rationality and sophistication are no longer considered part of masculinity, Orson Welles loses his highly masculine rating. Muhammed Ali is now considered the most masculine, with John Wayne next; surprisingly, Ralph Nader comes in third, before John Travolta and Jerry Lewis.

One method of quantifying the degree to which each dimension of the four-dimensional solution represents components taken from the three-dimensional solution is by means of a table of correlations between dimensions in the two solutions. Table C–21 gives such correlations. Here, however, we have replaced the unconstrained three-dimensional solution by its very similar constrained counterpart. This should not affect the conclusions much, because the two versions of the three-dimensional solution are so

TABLE C-19. Mode A for the Four-Dimensional Constrained Solution

	Mode A: Cars & Stars & Self			
	1	2	3	4
1	-0.87	-0.18	-0.28	-0.60
2	-0.34	-0.14	-1.33	1.45
3	-1.55	-0.04	0.80	2.80
4	-0.33	-0.53	2.53	-2.08
5	-0.25	-0.56	-2.40	1.31
6	-0.57	1.39	-1.39	0.40
7	-0.72	-0.58	0.92	0.62
8	-0.23	0.06	-0.60	-0.49
9	-0.13	-0.84	0.19	0.66
10	-0.02	-0.98	1.98	0.98
11	-0.11	-0.04	-1.03	0.05
12	-0.94	-0.72	0.49	-2.18
13	1.17	-0.32	0.01	-0.18
14	0.92	0.93	-0.14	-0.56
15	0.46	-0.61	0.18	-0.38
16	-1.11	2.54	-0.33	-0.26
17	0.23	0.78	-0.05	-0.38
18	2.31	-1.68	0.63	0.10
19	1.03	1.24	-.09	0.04
20	1.71	-0.95	0.19	0.12
21	2.19	-0.20	0.08	-0.19
22	-1.33	2.38	-0.49	-0.20
23	0.03	-0.38	0.40	-0.04
24	0.05	-0.02	0.46	-0.13
25	-0.87	-0.41	-0.73	-0.85

similar (for example, see Table C–11). The advantage of using the two constrained solutions is that it allows us to consider the Mode *B* part of the table, where dimensions are constrained to be orthogonal, in terms of additive variance components. For Mode *B* in Table C–21, correlations can be viewed as multiple regression weights, and the sums of squared correlations as variance-accounted-for values. We note that the row sums of the Mode *B* table all are equal to 1.0, indicating that all the variance of the three-dimensional solution is retained in the four-dimensional solution, only redistributed across the four dimensions as indicated by the squares of the correlations in the table. The column sums of squared correlations indicate the proportion of variance in each new dimension of the four-dimensional solution which can be predicted from the dimensions of the three-dimensional solution. Dimension one is almost entirely predictable from the three-dimensional solution (.98) and this is almost all from the old version of dimension one. Thus, dimension one can be considered unchanged. Dimension two is primarily composed of vari-

TABLE C-20. Mode B for the Four-Dimensional Constrained Solution

	Mode B: Rating Scales			
	1	2	3	4
1	0.69	-0.09	-0.42	1.82
2	0.65	-1.08	0.48	-1.30
3	-0.12	-1.99	0.34	0.46
4	1.31	0.99	-0.26	-0.77
5	-1.46	-0.07	-0.22	0.96
6	-1.79	0.08	0.12	0.63
7	1.21	-0.10	0.54	-0.99
8	-1.24	1.77	1.47	0.56
9	0.76	-1.23	0.42	0.43
10	1.09	0.21	-0.31	1.25
11	-1.20	0.05	-1.79	-0.82
12	-0.65	0.02	-2.34	-0.68
13	-0.42	-1.17	1.74	0.41
14	-0.76	-0.35	2.03	0.57
15	0.81	0.11	-0.15	0.15
16	1.16	0.19	-0.90	0.88
17	-0.73	0.33	-1.96	-0.89
18	1.09	0.46	0.27	-1.36
19	-0.13	0.13	0.37	-2.05
20	1.47	1.49	0.11	-1.03
21	0.88	-0.49	1.42	-0.37
22	-0.13	-0.68	0.19	-2.50
23	-1.55	-0.88	-0.03	0.34
24	1.35	-0.45	0.44	0.07
25	-1.02	0.30	0.25	-1.11
26	-1.01	1.02	-1.33	-0.28
27	0.36	0.30	0.43	1.26
28	-0.99	1.94	1.46	0.01
29	0.34	-1.26	-1.23	1.51
30	0.06	0.77	-0.74	-0.79
31	-1.58	0.23	-0.06	0.38
32	-0.80	-0.88	1.04	-0.07
33	-0.99	-0.72	0.02	1.11
34	1.04	-0.78	-1.67	0.57
35	0.55	1.72	1.12	0.32
36	0.07	2.15	-0.87	1.02
37	1.15	0.45	0.32	0.18
38	-0.21	-2.28	-0.13	-1.17
39	1.13	-0.21	-1.00	1.28

ance from the old dimension two, but 15% of its variance cannot be predicted from the old dimensions. Dimension three is the most "novel" dimension, since only 57% of its variance can be predicted by the dimensions of the old three-dimensional solution.

TABLE C-21. Splitting Process for Four-Dimensional Constrained Solution

		Mode A			
		4-D			
		1	2	3	4
3-D	1	.99	-.39	.02	-.07
	2	-.35	.93	-.56	.21
	3	.19	-.20	.73	-.84

		Mode B				
		4-D				
		1	2	3	4	Row Sum of Squares
3-D	1	.99	-.06	-.15	-.04	1.00
	2	-.03	.83	-.43	-.35	1.00
	3	.06	-.02	.60	-.79	1.00
Column Sum of Squares		.98	.85	.57	.75	

The correlations show that it combines aspects of both the old dimension two and dimension three. Finally, dimension four is primarily the same as the old masculinity-femininity dimension, but also contains some variance from the old dimension two. For this dimension, 25% of its Mode *B* loading variance cannot be predicted from the three-dimensional solution and hence may represent new shades of meaning for femininity.

We can conclude from this four-dimensional solution that our original "mature/conservative" and "feminine-soft-smooth" dimensions (from the three-dimensional solution) were not simply single overtones but rather were composed of several threads of connotation, closely associated. It is perhaps an unfortunate comment on the chauvinistic perspective of the MBA student sample used for this study that masculinity tended to be associated to some extent with rationality, seriousness, and sophistication, and consequently came out on the same dimension in the three-dimensional solution. However, our disapproval is mitigated by the fact that these two overtones were at least somewhat distinguishable to our subjects, as is demonstrated by the rearrangement that occurs in the four-dimensional solution.

FURTHER STEPS

The next step, after identifying the dimensions, is to use the information they provide for the purpose of evaluating congruence of automobiles and potential spokesmen. This is not simply a matter of examination of the distance between points in the stimulus space, but rather intelligent consideration of the nature of different overtones and how they might interact. A further step after that is to look at the person loadings and use them to examine market segment differences. These and other theoretical issues are discussed in the complete version of this article (Harshman and De Sarbo 1981).

NOTES

1. This appendix was excerpted from a longer manuscript, Harshman and De Sarbo (1981), which deals in more detail with the theoretical issues of the particular marketing application, the use of the dimensions for marketing decisions, and the relative merits of this approach compared to MDS.

2. This work was done at Bell Laboratories, Murray Hill, New Jersey, while Richard Harshman was on leave from the Department of Psychology, University of Western Ontario, London, Canada N6A 5C2. We are grateful to Bell Laboratories for the support that made this research possible.

3. Since three solutions were used for the full data, plus 3 solutions for each of the 4 split-halves (to be discussed below), there were in fact 15 solutions obtained at each dimensionality between two and six.

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