Models for Analysis of Asymmetrical Relationships Among N Objects or Stimuli

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Suppose that a sociologist who is studying a group of 20 individuals obtains, from an outside observer, a set of ratings of the amount of interpersonal attraction among members of the group. Let us assume that the result is a 20 by 20 matrix X, where a given cell, x_{ij} , contains the observer's rating of the amount that person i likes person j. How is such a matrix to be analyzed? Since it might be considered a type of proximity matrix, the data might seem to call for a multidimensional scaling analysis. There is a problem, however, in that "liking" is an asymmetrical relationship. The amount that person i likes person j need not equal, or even resemble, the amount that person j likes person i.

<u>Spatial models and symmetrized data</u>. Since multidimensional scaling is based on a spatial approach to modeling data, where the proximity of points in the model stands for the strength of relationship between variables in the original data, and since spatial relationships are intrinsically symmetric, the resulting representation is, of necessity, limited to describing the symmetrical aspects of the data. Furthermore, the same limitation will be encountered with other analysis techniques commonly applied to matrices of relationships among variables. Factor analysis, for example, can also be thought of as using a spatial model, where angles between vectors represent correlations between variables. Even with "nonspatial" techniques which use cluster and tree-structure models, intrinsically symmetrical representational schemes are usually employed,

e.g., by modelling data relationships in terms of "distance" between clusters, or along branches of trees (see, for example, discussion by Carroll, 1976). While some of these prcedures allow asymmetric data to be input, none of them represents asymmetrical aspects of the data in their output. Thus conventional techniques for analyzing tables of relationships seem inadequate for handling potentially important characteristics of certain types of relationship matrices.

Clearly, the limitations of the spatial models have not prevented their wide and successful application, even in some cases where the data is asymmetrical. Small asymmetries in a matrix of relationships can often be attributed to measurement error or other extraneous factors, and disregarded. It is common practice, for example, to prepare data for input to multidimensional scaling by averaging corresponding values on either side of the main diagonal.

Larger and more systematic asymmetries can sometimes be handled by attributing them to multiplicative row or column biases (Luce, 1959; Indow & Uchizono, 1960; see also Bishop, Fienberg & Holland, 1975, Ch. 8). Coombs (1964) makes a distinction between a symmetric proximity matrix and a class of nonsymmetric proximity matrices which he calls "conditional proximity matrices". He attributes the asymmetries in conditional proximity matrices to the fact that the different rows are not in a common scale, proximities in each row being relative only to others within that row. He assumes that such a matrix should first be subjected to separate adjustments to each row which will (hopefully) restore the matrix to approximate symmetry. Then, MDS-like techniques can be applied to the adjusted symmetrized matrix.

Intrinsic asymmetries. These approaches to asymmetrical data can be thought of as providing pre-processing which helps to uncover the "underlying symmetry" of the matrix. They may not be appropriate if the asymmetries are considered a basic property of the relationships under analysis. Sometimes, one is confronted with a square matrix of relationships among n things, where the type of relationship involved is considered intrinsically asymmetrical. The matrix of "liking" relationships is one such case. There seems no reason to attribute any observed "liking" asymmetries to row or column specific biases on the part of the rater, nor does Coombs' definition of a conditioned proximity matrix seem to apply (since the one rater presumably applied the same criteria for each row of his rating matrix). Instead, it seems that the nature of the relationship itself is fundamentally directional, and thus potentially asymmetrical. Other examples of essentially asymmetric relations might include word association data, where each cell of the matrix gives the relative frequency with which the row word as stimulus elicited the column word as response, or international trade data, where the number in each cell is the value of goods sold by the row nation to the column nation. In such cases, the asymmetries may provide important information about the processes involved, and consequently one is reluctant to artificially symmetrize the matrix.*

When the asymmetries are thought to be basic to the relations involved, any analysis which relies only on a spatial model of the relationships would

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* Sometimes, however, one may want to remove row or column biases from the data in order to separate any "artifactual" or conditional asymmetries from the "underlying" asymmetries of interest. It will shown below that additive or multiplicative row and column adjustments will not, in general, remove all the asymmetries from a matrix of more complex asymmetric structure, although such adjustments may absorb part of the "real" asymmetries of interest along with any "artifactual" asymmetries.

prove to be too restrictive; what is needed is a method which incorporates nonsymmetric relationships into the model itself in order to provide a solution which can represent any meaningful patterns of nonsymmetry which may be present in the original data. While some recently proposed methods of dealing with asymmetric data have employed a spatial model which is modified or "enriched" to incorporate a nonspatial component (e.g., Tobler, 1976, to be discussed below) the model to be presented in this article goes even further and adopts a completely nonspatial approach. It is suggested that this approach might prove useful for analysis of certain asymmetrical matrices where the observed relationships can be conceived as arising from combinations of underlying component relations which are themselves basically asymmetric.

The DEDICOM Models

Instead of representing observed relationships between objects (e.g., strength of liking between persons) in terms of a simplified model which involves spatial relationships (e.g., proximities of points), let us consider an alternative approach which would represent the observed relationships in terms of a simplified model which involves relationships <u>of the same kind as occur in the observed data</u>, but defined in terms of a few "higher level" objects (or stimulus types or dimensions) rather than the observed objects (e.g., strength of liking between <u>types</u> of persons). Each observed directional relationship would then be decomposed into some combination of these few basic underlying relationships. Since the underlying relationships are also directional (because they are the same kind as the surface data relationships), this type of representational procedure will be called DEDICOM, for <u>DE</u>composition into <u>DI</u>rectional <u>COM</u>ponents. In this article we will consider a family of related models which embody the DEDICOM approach.

The single domain ("strong") model. Let X represent the n by n matrix of observed asymmetrical relationships, so that an element of X, e.g., x_{ij}, represents the directed relationship of object i to object j. The simplest version of the DEDICOM model would represent this data as follows:

(1) $X = A\ R\ A' + E$,

where A is an n by q (vertical) matrix of "loadings" of the n objects on a few basic "types" of objects. R is a small, usually asymmetric, q by q matrix giving the directional relationships among the basic q "types" or dimensions, and E is simply a matrix of error terms or residuals of prediction (characteristics of E will be discussed in a later section).

The general method of interpreting this model can be demonstrated by referring back to the example of a "friendship" matrix, where x_{ij} would express the degree to which person i likes person j. For discussion, let's consider a 5-dimensional solution. A DEDICOM analysis would describe five basic factors or dimensions underlying the patterns of "liking" behavior for these individuals. These dimensions might be thought of as five idealized "types" of individuals, types which each real individual resembles to various degrees. Thes dimensions would be named in the same way as factors in a Q-type factor analysis (i.e., by examining A to see which individuals had high loadings on each dimension). For example, if the individuals involved were members of an all-male high school class, a label of "athletic type" might be assigned to the dimension on which students active in sports had high loadings. Other dimensions such as "student government type", "intellectual type", "socialite type" and "rowdy-tough type" might emerge from the analysis. (Note that these dimensions do not necessarily assign individuals exclusively to a given "type". Unlike a cluster analysis, DEDICOM allows individuals to have moderate loadings on any given dimension, and high loadings on more than one dimension. For example, an individual can be high on both "athlete type" and "student government type"; such an individual would show liking patterns which resembled those of both these types of students.) Alternatively, one might think of the dimensions as personality traits ("athleticism", etc.) and the loadings as describing how much of each trait the person had.

The R matrix would not relate to specific individuals, but instead would provide a general statement of the patterns of "liking" among the five types of students. Each row of the R matrix would describe how much a given type of student generally tends to like students of the other four types (and the same type, if the diagonal element is considered). For example, r_{st} might be the amount that student government types generally tend to like the athlete types, and r_{ts} would give the converse relation, how much the athlete types tend to like the student government types. (Alternatively, if the dimensions are thought of as personality traits, each element of the R matrix would describe how much the presence of a given personality trait in an individual generally tends to make that person attracted to individuals possessing another particular personality trait.)

To clarify how the observed "liking" values in X are built from these hypothesized underlying attractions described by R, it is helpful to refer to the scalar form of equation (1), namely

(2) $$x_{ij} = \sum_{s=1}^{q} \sum_{t=1}^{q} a_{is} r_{st} a_{jt} + e_{ij} \quad .$$

The degree to which person i likes person j is the sum of q^2 components (5^2 for the example considered above), and each component is computed by means of a triple ptoduct. Each triple product expresses a directional link between one "type" component of person i and one "type" component of person j. This link is weighted by the amounts that the individuals actually resemble the types in question. For example, in one triple product, a_{is} would represent how much the i^{th} person is similar to the athlete type, r_{st} would indicate how much the athlete type generally likes the student government type, and a_{jt} would represent how much the j^{th} person resembles the student government type. This product thus expresses how much the athlete component of i likes the student government component of j. As a result of the double summation in (2), the model accumulates the attractions due to the amount that each component of person i likes each component of j. The composite liking which results from this sum approximates the observed overall amount that i likes j. (If we consider the alternative formulation in terms of personality traits, we would describe the triple product as multiplying the amount of the athlete trait that person i has times the amount that individuals with the athlete trait generally like others with the student government trait, times the amount of the student government trait that person j has.)

We can now see the sense in which the surface directional relationships are decomposed into underlying directional components which are of the same kind as the surface ones. Where the observed data is a table of asymmetrical feelings of liking among individuals, the reduced rank representation or model also involves asymmetrical feelings of liking. In the model, however, these feelings hold between the small number of components or "types" of individuals indicated by the analysis. Thus there are far fewer asymmetrical relationships to interpret, and these few relationships, given by the R matrix, will hopefully provide insight into the basic patterns of asymmetry underlying the larger matrix of observed values. From this representation, one can reconstruct (approximately) the observed asymmetrical relation between any two data sources (e.g., persons) by decomposing each source into its underlying components and summing the asymmetrical relations between the components of one and the components of the other.

To illustrate this decomposition, Table 1 gives a hypothetical matrix of friendship relationships among 20 members of a high school graduating class. Table 2 gives a DEDICOM analysis of this matrix into friendship patterns among five personality components or "types" of individuals. Table 3 gives the A matrix of loadings of each of the 20 individuals on the five basic "types".

It is apparent from equation (1) that the DEDICOM model is subject to the same kind of rotational indeterminacy as is two-way factor analysis and multidimensional scaling. The matrix A can be subjected to any general linear transformation provided the inverse transformation is applied on either side of the R matrix.

The dual domain ("weak") model. We have been considering a model which describes asymmetric relationships among objects in terms of asymmetric relationships among a single set of underlying components of the objects. In some circumstances, however, we may wish to consider a more general model, where there are two sets of underlying components, and the directional relationships are hypothesized to hold from components of one kind to components of the other kind. These two approaches have been discussed and compared by Coombs (1964, p. 426) in the context of conditional proximity matrices, and a similar distinction can be made in the context of intrinsically asymmetrical relationships.

Consider, for example, a matrix describing the visual gazing behavior of members of a psychotherapy group. In this matrix, x_{ij} might give the amount that person i looked at person j during the session. The first approach, which we will call the strong form of the model, would identify a single set of components or characteristics of people (given in the A matrix of (1)), and provide an R matrix that describes the general effect of possessing any given component on an individual's tendency to look at others with particular components. (Another way of putting this is to say that R would describe how much each "type" of person tended to look at each of the other "types".)

It might be argued, however, that the aspects of an individual that underly his patterns of looking behavior were of a different kind than those aspects of an individual that underly his tendency to be looked at. Looking behavior would be influenced by cognitive and motivational factors of the individual, whereas the tendency to be looked at would be influenced by visual, vocal, gentural, and other perceptual stimulus-like characteristics of the individual. (At least, such superficial characteristics might be important in initial therapy sessions, until the members of the group began to know one another better.) Thus the dimensions underlying the rows, the lookers, might be largely different from the dimensions underyling the columns, the looked-at. Even though the same objects comprise the rows as the columns, different characteristics of those objects may be important in their row roles than are important in their column roles.

Such a situation could be represented within the framework of the single domain DEDICOM model given by (1), but the representation would be cumbersome. One would construct an A matrix which was partitioned into two parts. The first set of columsn (call this submatrix $\overset{\circ}{A}$) might describe the dimensions of individuals which influenced their looking behavior; the second set of columns (call this submatrix $\overset{*}{A}$), might describe the dimensions of the individuals which characterize their properties as perceptual stimuli to others. The hypothesis that the looking behavior is mainly controlled by the directed relationships from the first kind of dimension to the second would be supported if the R matrix then took on a special form. Here, entries would be very small for directed relationships in R which relate the columns of $\overset{\circ}{A}$ to other columns of $\overset{\circ}{A}$, and much larger for relationships from the columns $\overset{\circ}{A}$ to the columns of $\overset{*}{A}$. Further, all entries in R for relationships from $\overset{*}{A}$ to any dimensions would be small. If we let the small effects vanish, then our model would take on the special form

$$(3) \qquad A = \left[\overset{\circ}{A} \;\middle|\; \overset{*}{A}\right]$$

$$(4) \qquad R = \left[\begin{array}{c|c} 0 & \dot{R} \\ \hline 0 & 0 \end{array}\right]$$

(5) $$X = \left[\overset{o}{A} \mid \overset{*}{A}\right] \left[\begin{array}{c|c} 0 & \dot{R} \\ \hline 0 & 0 \end{array}\right] \left[\begin{array}{c} \overset{o}{A} \\ \hline \overset{*}{A}' \end{array}\right] + E \quad ,$$

where $\overset{o}{A}$ and $\overset{*}{A}$ are n by (1/2)q. In this model, the R matrix is singular. The same psychological state of affairs can be less combersomely represented by a model which is both more compact and which also avoids the difficulties of singular matrices, if we simply relax the restriction that the left hand A matrix have the same for as the right hand matrix. This less constrained model, called the weak form of the DEDICOM representation, can be written as follows

(6) $$X = \overset{o}{A}\,\dot{R}\,\overset{*}{A}' + E \quad ,$$

where $\overset{o}{A}$ and $\overset{*}{A}$ are n by $\dot{q}$ and $\dot{R}$ is $\dot{q}$ by $\dot{q}$ with E, as usual, an n by n matrix of error terms. (Here $\dot{q}$ is $\frac{1}{2}$ q.)

In contrast with the strong DEDICOM model, this representation provides <u>two</u> sets of underlying components: $\overset{o}{A}$ gives the components of the objects in their row position and $\overset{*}{A}$ gives the components of the objects in their column position. The $\dot{R}$ matrix then gives the directed relationships from the $\overset{o}{A}$ components to the $\overset{*}{A}$ components.

We can apply this model to our hypothetical example of "looking" scores; to predict a specific relationship in X, one would multiply the amount of each motivational component possessed by the looker (as given in $\overset{o}{A}$) times the amount that the component generally prompts an individual to look at a particular stimulus characteristic (from $\dot{R}$) times the amount of that stimulus characteristics possessed by the person being looked at (given in $\overset{*}{A}$). By adding up these triple products across all types of motivational components and all types of stimulus components, the total tendency for one particular person to look at another could be estimated.

<u>Nonequivalence of strong and weak models</u>. It can easily be shown that when $\overset{*}{A}$ and $\overset{o}{A}$ are not linear transformations of one another, then there generally exists no solution of the form given in (1) for data fit by (3) unless one goes to a higher dimensionality (a proof of this is given in the Appendix). Consequently, the model of equation (1) makes a strong claim about a given data set, one which might have interesting substantive implications if verified. When the row dimensions and the column dimensions of a given asymmetrical relationship matrix can be demonstrated to span the same space, this agreement is a fact unlikely to arise by chance and probably demonstrates the existence of some common process underlying the row and column patterns of interaction. This leads naturally to an interpretation in terms of a single set of components which generated the relationships and thus which is being "uncovered" by the analysis in the form of equation (1). With data containing noise, of course, the row space and columnn space will probably not match exactly, but close agreement might still be interpreted as surprising and interesting. We will not discuss statistical tests of the fit of the two models in this article, but will demonstrate comparisons of the two models fit to a given set of real data (below).

The weak DEDICOM procedure can be seen as an application to asymmetric relationships of the principles developed for profile data by Tucker in his generalization of two-way factor analysis (as described in Levin, 1965). In both approaches the data is represented in terms of three matrices:

one matrix describing idealised row elements, another describing idealised column elements, and a third matrix which gives the relations between the idealised row elements and the idealised column elements.

Three-way versions. The models described above are easily generalized to three-way arrays consisting of a stack of m different n-by-n matrices of asymmetric relationships. Such m by n by n arrays might arise in various ways. If the data concerned feelings of liking among n persons, then successive layers of the stack might describe ratings for liking among the same n individuals on m different years. If the n by n asymmetric matrix gave word association frequencies, the different matrices might provide the associative strengths for different populations (young vs. old, males vs. females, etc.). One straightforward way to generalize the strong DEDICOM model in order to take into account a third mode would be as follows:

(7) $$X_i = A\,D_i\,R\,D_i\,A' + E_i \quad ,$$

where the D_i matrix is a diagonal matrix giving the weights for the underlying components for the i^{th} occasion. This representation of a three-way array is closely related to Harshman's (1972) generalization of the INDSCAL-PARAFAC three-way multidimensional scaling model, called PARAFAC2. In PARAFAC2, a set of covariance or cross-product matrices (X_i) is represented by an equation identical to (7), except that R is necessarily symmetrical, since the X_i are symmetrical. In PARAFAC2 R is interpreted as a matrix of cosines among dimensions (when appropriately normalized). If one applies the model to asymmetrical X_i matrices, with the understanding that R is allowed to be asymmetrical, then the three-way DEDICOM model given in (7) results. One of the reasons for interest in PARAFAC2 is that it is thought to have interesting uniqueness properties related to those of INDSCAL and PARAFAC1. No investigation of the uniqueness of three-way DEDICOM has been undertaken, but a model slightly more general than (7) has been shown to provide a "unique" solution. If the X matrices represent the asymmetrical relationships from the objects in one condition to the objects in another condition, then we might wish to consider classifying the asymmetrical relationship matrices in a doubly suscripted fashion, where X_{ij} represents the asymmetrical relationships from the objects when in condition i to the objects in condition j. A slightly generalized three-way model would then be appropriate, i.e.,

$$(8) \quad X_{ij} = A\, D_i\, R\, D_j\, A' + E_{ij} \quad .$$

This model differs from (7) only in that the right hand diagonal matrix differs from the left hand one (in a pattern determined by i and j). Harshman has recently shown (ref. Note 1) that this model provides a unique representation of the X_{ij} under certain minimal conditions provided one has an X_{ii} and an X_{ij} (or X_{ij} and X_{ik}) matrix in the set. This uniqueness could make these three-way versions of DEDICOM quite useful for identifying dimensions, when the data fulfills the appropriate requirements.

The dual domain ("weak") DEDICOM model can be similarly extended to three-way arrays, for example

$$(9) \quad X_i = \overset{o}{A}\, D_i\, \dot{R}\, D_i\, \overset{*}{A}' + E_i \quad ,$$

$$(10) \quad X_{ij} = \overset{o}{A}\, D_i\, \dot{R}\, D_j\, \overset{*}{A}' + E_{ij} \quad .$$

Similar comments about uniqueness apply to these dual domain models.

Ignoring the diagonal of X. For many types of relationship matrices, the diagonal values are either missing or undefined. Even when they are present, the "self-relationship" values given in the diagonal cells are often not comparable to the "inter-relationship" values in the off diagonal cells. A case in point is the hypothetical example of a "liking" matrix. In this matrix, the diagonal cells would correspond to a rating of how much the i^{th} person likes the i^{th} person, i.e., how much each person likes himself. A rater concerned with friendships would probably not be requested to rate "self liking", but if he were, his ratings would presumably be based on very different behavioral criteria than interpersonal liking, and thus would probably be measured on a different scale. Furthermore, even if a common scale of measurement could be defined, the psychological variables underlying self-liking would probably differ from those affecting interpersonal liking. For such data, it would not seem appropriate to fit a common model to both the diagonal and off-diagonal cells. And similar problems arise with the diagonal cells in matrices describing word associations and international trade. In such cases equations (1) and (2), should be interpreted as applying only to off-diagonal cells of X, and the diagonal values of the R matrix should be interpreted as giving the "within type" inter-relationships rather than a given type's self-relationships. (For example, diagonal R values would be interpereted as describing how much the members of a given "type" tend to like other members of the same type, rather than as giving the amount that individuals of a given type like themselves.)

The relative merits of including vs. ignoring the diagonal data cells have been explored with some real data sets. These results seem to support the idea that the diagonals of some data sets should be ignored.

Estimation

A. Dual Domain ("weak") DEDICOM

1. Take singular value decomposition of X

(11) $X = PDQ'$

2. For a q dimensional least squares solution, take first q singular vectors and rotate to desired pattern (e.g., simple structure)

(12) $\mathring{A} = P_q \mathring{T}$, or row standardized $D P_q \mathring{T}_2$

$\overset{*}{A} = Q_q \overset{*}{T}$ or, row standardized, $D Q_q \overset{*}{T}_2$

3. Estimate R by least squares, given A

(13) $R = \mathring{A}^{+} X (\overset{*}{A}^{+})'$ (where $\mathring{A}^{+} = (\mathring{A}'\mathring{A})^{-1} \mathring{A}'$)

4. For factor-like (uncorrelated errors) solution, use XX' in line 1, and iterate on the diagonal to estimate diagonals of XX'.

B. Single Domain ("strong") DEDICOM

1. Closed form approximate least squares solution

(a) produce a symmetric transformation of X which incorporates its row space and column space, appropriately weighted, e.g.,

(i) rotation method -- take singular value decomposition of X

(14) $X = PDQ'$

define the symmetrizing rotation L

(15) $L = QP'$

produce V, the common-space transformation of X

(16) $V = XL + LX = PDP' + QDQ'$

(ii) cross-product method

(17) $V = XX' + X'X = PD^2 P' + QD^2 Q'$

(b) Take singular value decomposition of V to get A

(18) $V = \overset{*}{P}\overset{*}{D}\overset{*}{Q}'$

take first q singular vectors for (approximate) least squares solution in q dimensions, rotate to desired pattern (e.g., single structure)

(19) $A = \overset{*}{P}_q T$ or, row standardized, $D\overset{*}{P}_q T_2$

(c) Estimate R by regression methods

(20) $R = A^+ X(A^+)'$

2. <u>ITERATIVE Solution</u>

for n^{th} iteration:

(21) $A_{Left} = X A^{+\prime}_{(n-1)} R^{-1}_{(n-1)}$

(22) $A_{Right} = (R^{-1} A^{+}_{(n-1)} X)'$

(23) $A_n = \frac{1}{2} (A_{Left} + A_{Right})$

(24) $R_n = A^+_n X(A^+_n)'$

3. <u>True Least Squares Solution</u>

Use function minimization procedure (currently employing IMSL Z X MIN quasi newton method) to find A, R matrices which minimize sum of squared errors of fit. Bill Krane (U.W.O.) has worked out the derivatives for A, R, so that an efficient minimization algorithm is anticipated.

4. Skew Symmetric Special Case

For skew symmetric matrices, the row and column spaces are the same, so the method employed for the weak model will provide a single domain ("strong") solution. Furthermore, taking the skew symmetric part of X will not change A if all the dimensions in A have skew-symmetric components to their relations in R. In other words,

$$(25) \quad (X - X') = A\,(R - R')A' + (E - E')$$

5. Three-way DEDICOM

One may use a NILES type of successive regression procedure to fit the three-way models to data. This makes three-way DEDICOM a generalization of PARAFAC2. Unique solutions may result.

6. Factor-like or Component-like Solutions

We have been considering a component-analysis or MDS-analysis-like procedure. If one wants to consider more carefully the appropriate model for E in the equation $X = ARA' + E$ one may wish to specify that the rows and columns of E are uncorrelated. This corresponds to the assumption that the observations generating X represent an estimate of some underlying structure ARA' perturbed by random measurement error E. Estimation procedures for A, R under those assumptions would then yield what might be considered a generalization of the common-factor model where Φ, the factor correlation matrix, was allowed to be asymmetric. It may be that this could be accomplished by factoring $XX' + X'X$, but these questions have not yet been investigated.

Coombs (1964) Journal Citation Data

Number of References in Row Journal to Column Journal

122.	4.	1.	23.	4.	2.	135.	17.	39.	1.
23.	303.	9.	11.	49.	4.	55.	50.	48.	7.
0.	28.	84.	2.	11.	6.	15.	23.	8.	13.
36.	10.	4.	304.	0.	0.	98.	21.	65.	4.
6.	93.	11.	1.	186.	6.	7.	30.	10.	14.
6.	12.	11.	1.	7.	34.	24.	16.	7.	14.
65.	15.	3.	33.	3.	3.	337.	40.	59.	14.
47.	108.	16.	81.	130.	14.	193.	52.	31.	12.
22.	40.	2.	29.	8.	1.	97.	39.	107.	13.
2.	0.	2.	0.	0.	1.	6.	14.	5.	59.

VARIANCES OF THE ORIGINAL DATA USED IN THIS ANALYSIS

VARIANCES INCLUDING DIAGONAL ELEMENTS
TOTAL= 3879.33560
SYMMETRIC PART= 3581.76560
ASYMMETRIC PART= 297.570000

Simple A

OBLIQUE A

0	1.000	2.000
1 AM J PSYCH	.288	.002
2 J AB SOPSY	-.013	.829
3 J AP PSYCH	.017	.086
4 JCOMPHYPSY	.509	-.156
5 J CON PSYC	-.032	.449
6 J ED PSYCH	.029	.039
7 J EX PSYCH	.734	.020
8 PSYCH BULL	.241	.262
9 PSYCH REVU	.242	.095
10 PSYCOMTRKA	.028	.029

R MATRIX

0	1.000	2.000
1 RMTX	484.281	58.324
2 RMTX	100.584	373.866

Rescaled A

0	1.000	2.000
1 AM J PSYCH	.440	.004
2 J AB SOPSY	-.026	.738
3 J AP PSYCH	.033	.171
4 JCOMPHYPSY	.560	-.156
5 J CON PSYC	-.052	.546
6 J ED PSYCH	.087	.114
7 J EX PSYCH	.547	.015
8 PSYCH BULL	.247	.257
9 PSYCH REVU	.338	.126
10 PSYCOMTRKA	.071	.069

R MATRIX

0	1.000	2.000
1 RMTX	31.870	5.329
2 RMTX	6.803	28.021

CORRELATION (INCLUDING ALL CELLS)= .78905001
CORRELATION (EXCLUDING DIAGONAL)= .73451722
CORRELATION (DIAGONAL ONLY)= .90036404

Coombs Journal Citation Data -- Symmetrizing transformations   V = XL + LX

XL PART OF V

0		1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
1	AM J PSYCH	137.042	12.143	1.199	35.376	-3.006	7.564	103.605	56.191	36.687	1.187
2	J AB SOPSY	12.143	294.658	23.301	10.777	65.380	10.780	22.478	90.622	45.776	1.182
3	J AP PSYCH	1.199	23.301	86.867	2.777	12.024	12.049	8.372	19.900	9.282	6.926
4	JCOMPHYPSY	35.376	10.777	2.777	308.972	-10.218	1.753	52.839	80.334	43.819	1.487
5	J CON PSYC	-3.006	65.380	12.024	-10.218	170.135	4.911	-19.752	103.091	12.014	4.402
6	J ED PSYCH	7.564	10.780	12.049	1.753	4.911	57.211	16.527	19.695	10.030	9.586
7	J EX PSYCH	103.605	22.478	8.372	52.839	-19.752	16.527	291.212	136.964	77.664	7.207
8	PSYCH BULL	56.191	90.622	19.900	80.334	103.091	19.695	136.964	171.231	39.056	2.137
9	PSYCH REVU	36.687	45.776	9.282	43.819	12.014	10.030	77.664	39.056	110.535	8.623
10	PSYCOMTRKA	1.187	1.182	6.926	1.487	4.402	9.586	7.207	2.137	8.623	58.753

LX PART OF V

0		1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
1	AM J PSYCH	113.748	17.421	.156	27.507	10.564	2.211	93.066	13.406	27.298	1.236
2	J AB SOPSY	17.421	317.030	13.979	13.569	80.364	6.063	51.235	53.725	41.677	7.509
3	J AP PSYCH	.156	13.979	82.610	4.026	11.417	6.397	8.414	18.094	1.653	7.797
4	JCOMPHYPSY	27.507	13.569	4.026	302.738	12.786	.943	76.781	18.516	50.471	2.457
5	J CON PSYC	10.564	80.364	11.417	12.786	211.087	8.866	37.539	33.485	3.433	10.663
6	J ED PSYCH	2.211	6.063	6.397	.943	8.866	33.137	8.401	10.040	-.746	4.785
7	J EX PSYCH	93.066	51.235	8.414	76.781	37.539	8.401	404.305	58.493	78.154	15.574
8	PSYCH BULL	13.406	53.725	18.094	18.516	33.485	10.040	58.493	32.547	34.241	18.010
9	PSYCH REVU	27.298	41.677	1.653	50.471	3.433	-.746	78.154	34.241	108.510	8.916
10	PSYCOMTRKA	1.236	7.509	7.797	2.457	10.663	4.785	15.574	18.010	8.916	60.904

EIGENVALUES (IN DESCENDING ORDER)=
528.584   333.887   262.279   165.079   91.1328   85.0320   78.2854   56.6966

DIAGONAL SCALE FACTORS=
.380070   .235693   .525890   .271465   .322698   .779960   .185261   .259349

37.9308   27.7094

.348939   .659041

Three- and Four-Dimension Analysis of Coombs (1964) Journal Citation Data

OBLIQUE A

0		1.000	2.000	3.000	1.000	2.000	3.000
1	AM J PSYCH	.328	-.013	.081	.542	-.044	.002
2	J AB SOPSY	-.036	.838	-.019	-.035	.750	-.016
3	J AP PSYCH	.015	.087	.004	.035	.171	.005
4	JCOMPHYPSY	-.023	-.012	.988	-.018	-.010	.985
5	J CON PSYC	-.042	.454	-.021	-.058	.558	-.024
6	J ED PSYCH	.037	.036	-.007	.124	.101	-.025
7	J EX PSYCH	.892	-.033	-.034	.718	-.057	-.059
8	PSYCH BULL	.218	.266	.081	.246	.250	.074
9	PSYCH REVU	.208	.102	.118	.316	.120	.135
10	PSYCOMTRKA	.034	.027	-.003	.094	.060	-.011

R MATRIX

0		1.000	2.000	3.000	1.000	2.000	3.000
1	RMTX	426.076	49.249	56.295	26.356	5.191	4.761
2	RMTX	110.026	375.956	30.757	7.687	27.748	2.030
3	RMTX	131.701	25.021	314.446	8.681	1.760	23.263

CORRELATION (INCLUDING ALL CELLS)= .89654211
CORRELATION (EXCLUDING DIAGONAL)= .81458784
CORRELATION (DIAGONAL ONLY)= .95664195

OBLIQUE A

0		1.000	2.000	3.000	4.000	1.000	2.000	3.000	4.000
1	AM J PSYCH	.327	-.005	.081	-.025	.540	-.039	.002	-.044
2	J AB SOPSY	-.025	.980	-.015	-.013	-.029	.965	-.021	-.015
3	J AP PSYCH	.016	.046	.004	.092	.040	.100	.007	.149
4	JCOMPHYPSY	-.023	-.015	.987	-.010	-.018	-.019	.986	-.004
5	J CON PSYC	-.034	-.020	-.016	.931	-.028	-.019	-.006	.935
6	J ED PSYCH	.037	.012	-.006	.051	.128	.031	-.025	.120
7	J EX PSYCH	.892	-.035	-.034	-.030	.717	-.068	-.060	-.036
8	PSYCH BULL	.222	.107	.093	.336	.256	.103	.078	.264
9	PSYCH REVU	.209	.152	.119	-.065	.316	.202	.135	-.079
10	PSYCOMTRKA	.034	-.005	-.002	.060	.098	-.018	-.009	.117

R MATRIX

0		1.000	2.000	3.000	4.000	1.000	2.000	3.000	4.000
1	RMTX	428.287	40.773	57.027	85.798	26.689	3.945	4.858	5.55
2	RMTX	93.749	319.804	23.252	66.375	6.565	18.830	1.721	5.52
3	RMTX	132.687	23.809	314.766	6.144	8.756	1.898	23.273	.27
4	RMTX	54.502	117.201	20.114	216.317	4.092	8.901	1.006	22.94

CORRELATION (INCLUDING ALL CELLS)= .93531130
CORRELATION (EXCLUDING DIAGONAL)= .86482781
CORRELATION (DIAGONAL ONLY)= .97134955

Strong DEDICOM analysis of Townsend, 1971, alphabetic confusion matrix (Condition 1)

Raw Data Analysis

OBLIQUE A

0		1.000	2.000	3.000	4.000
1	A	-.033	.078	.171	.110
2	B	.099	.015	-.017	.125
3	C	.285	.061	.020	-.062
4	D	.291	-.008	.000	.016
5	E	.057	.163	.028	-.001
6	F	.010	.233	.019	.016
7	G	.324	.006	-.018	.020
8	H	.075	.024	-.014	.428
9	I	-.054	.584	-.068	-.042
10	J	.015	.291	.108	-.016
11	K	.006	.125	.165	.133
12	L	.012	.517	-.141	-.024
13	M	-.112	-.052	-.170	.536
14	N	-.005	-.047	-.036	.479
15	O	.657	-.019	.015	-.034
16	P	.151	.125	.003	.075
17	Q	.417	-.049	-.005	-.018
18	R	.084	.070	-.038	.231
19	S	.054	.063	.062	.059
20	T	.017	.367	.056	.016
21	U	.211	-.027	.039	.211
22	V	-.049	.028	.274	.194
23	W	-.002	-.030	.050	.265
24	X	-.076	.041	.386	.119
25	Y	-.032	.152	.257	.093
26	Z	.022	-.073	.756	-.089

R MATRIX

0	1.000	2.000	3.000	4.000
1 RMTX	78.476	8.113	2.946	13.171
2 RMTX	6.228	88.628	3.729	7.003
3 RMTX	5.288	10.353	71.933	6.957
4 RMTX	12.174	7.706	4.714	83.654

Skew Symmetric Data Analysis

OBLIQUE A

0		1.000	2.000	3.000	4.000
1	A	.008	.043	-.138	.206
2	B	-.164	.359	-.059	.012
3	C	.050	-.016	-.149	.216
4	D	-.155	.306	.092	-.044
5	E	-.021	.071	-.202	.345
6	F	-.083	-.137	.058	.517
7	G	.196	.281	-.242	.037
8	H	.548	-.061	.150	.086
9	I	.041	.070	.335	-.259
10	J	-.004	.058	.248	.229
11	K	.061	.100	.226	.197
12	L	.080	.010	.367	.179
13	M	.011	.193	.014	.016
14	N	.148	-.167	.055	-.009
15	O	.618	-.025	-.102	-.167
16	P	.137	.110	-.037	.066
17	Q	-.021	.598	.046	-.147
18	R	.262	.176	-.053	.173
19	S	-.063	.229	-.114	.148
20	T	-.034	.002	.588	.021
21	U	.285	.238	-.007	.076
22	V	.018	.070	.076	.099
23	W	-.017	.181	-.052	.075
24	X	.069	.037	.093	.240
25	Y	.012	.191	.256	-.011
26	Z	-.066	.026	-.012	.156

R MATRIX

0	1.000	2.000	3.000	4.000
1 RMTX	-.000	-16.541	-.492	-3.708
2 RMTX	16.541	.000	1.299	.336
3 RMTX	.492	-1.299	.000	-13.975
4 RMTX	3.708	-.336	13.975	.000

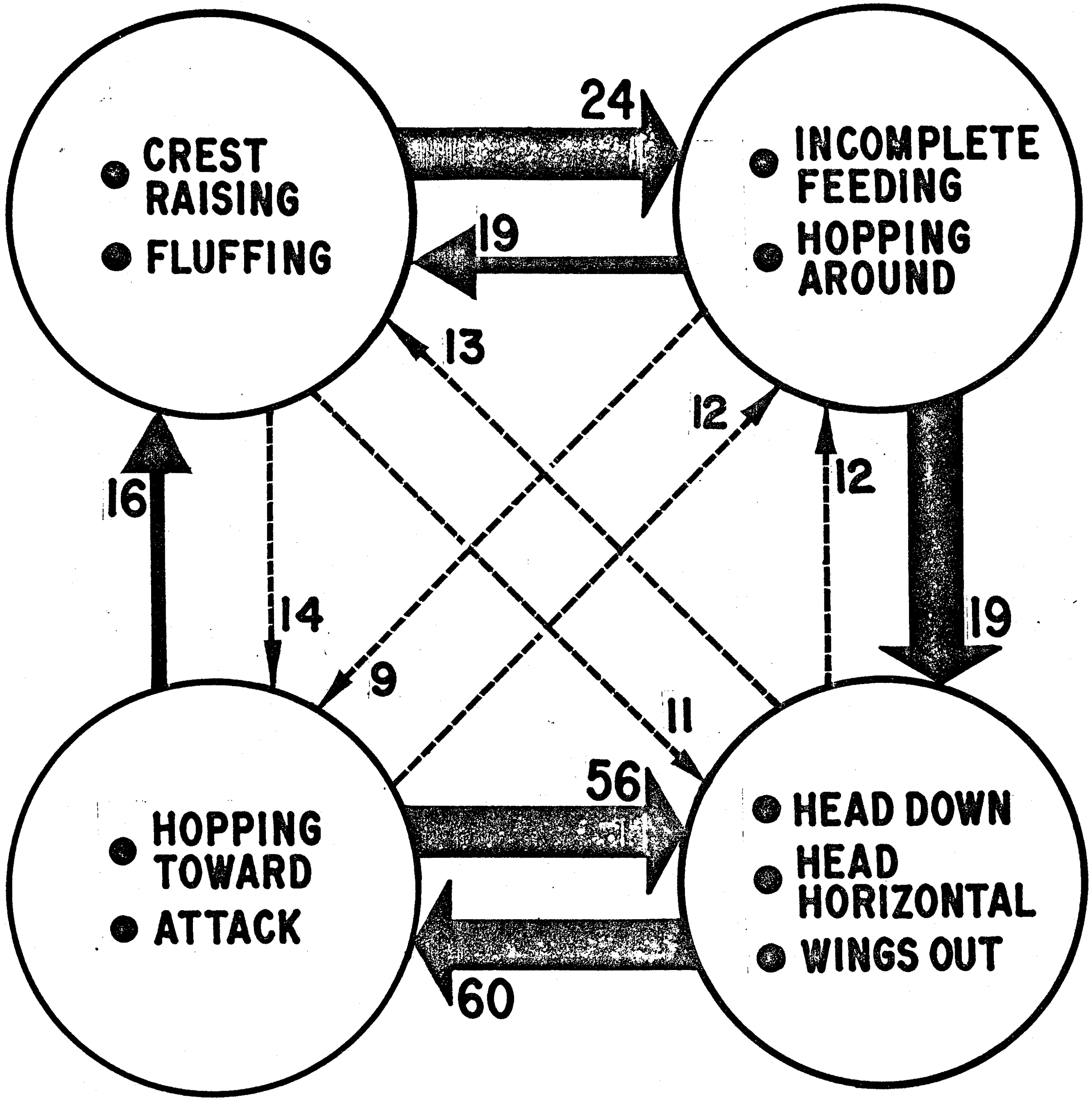
Eight Dimensional DEDICOM analysis of skew symmetric part of Townsend data

OBLIQUE A

0		1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
1	A	.058	-.028	-.057	.220	.235	.117	-.121	.020
2	B	-.048	-.026	-.027	.038	-.027	.864	.039	.034
3	C	-.114	-.070	-.190	.107	.109	-.047	.115	.103
4	D	-.109	.335	.058	.028	.037	.265	-.011	-.112
5	E	-.126	.088	-.256	.230	-.067	-.225	.054	.313
6	F	.042	-.034	.038	.758	-.020	.053	.002	-.036
7	G	.275	-.120	-.013	-.114	.168	.166	-.195	.528
8	H	.131	-.087	-.051	-.019	.086	.058	.737	-.078
9	I	-.035	-.040	.359	-.322	-.037	.035	.068	.034
10	J	.076	.008	.331	.289	.102	.080	-.077	.052
11	K	-.105	-.016	.185	.051	-.142	-.103	.203	.334
12	L	-.038	.016	.263	.172	-.163	.033	.313	.038
13	M	-.102	.143	-.008	-.076	.040	-.014	.069	.144
14	N	-.011	.014	-.007	.045	.350	-.066	.142	-.414
15	O	.884	.073	-.002	.032	-.043	-.049	.073	.028
16	P	-.018	.125	-.169	.011	-.114	.023	.319	.081
17	Q	.078	.876	-.007	-.020	.024	-.032	-.044	-.014
18	R	-.065	.040	.036	-.014	.755	-.036	.045	.026
19	S	-.090	.072	-.046	.030	.079	.016	-.111	.315
20	T	-.010	-.003	.670	.052	.051	-.076	-.070	-.013
21	U	.011	.007	.025	-.138	.290	.024	.199	.285
22	V	-.093	.050	.039	.043	.040	-.012	.107	.058
23	W	-.091	.077	-.081	-.002	-.026	.112	.086	.162
24	X	-.065	.001	.105	.134	.103	-.182	.053	.204
25	Y	-.070	.150	.208	-.062	-.125	.025	.155	.137
26	Z	-.044	.052	-.018	.163	-.028	-.032	-.031	.062

R MATRIX

0		1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
1	RMTX	.000	-11.466	-2.270	.174	-.767	-1.481	-3.860	-3.193
2	RMTX	11.466	.000	.644	-.521	1.923	-.872	3.780	3.315
3	RMTX	2.270	-.644	-.000	-12.290	-1.859	-.474	-1.067	-3.248
4	RMTX	-.174	.521	12.290	.000	-.059	.990	1.903	.210
5	RMTX	.767	-1.923	1.859	.059	.000	-9.679	2.702	-.790
6	RMTX	1.481	.872	.474	-.990	9.679	-.000	3.646	.771
7	RMTX	3.860	-3.780	1.067	-1.903	-2.702	-3.646	-.000	-11.463
8	RMTX	3.193	-3.315	3.248	-.210	.790	-.771	11.463	-.000



Graph of R matrix relations from DEDICOM analysis of <u>Parus Major</u> preceeding-following behavior matrix

- from Spence, 1978 (original data from Blurton Jones, 1968)